36 $\lim_{x\to 2} \frac{1}{x} = \frac{1}{2}$ use ε–δ to explain why...

Let $\varepsilon > 0$ be arbitrary and choose $\delta = \min\left(\frac{1}{2}, 3\varepsilon\right)$

Then $0 < |x-2| < \delta$ implies $-\delta < x-2 < \delta$

$2-\delta < x < 2+\delta$

since $\delta \le \frac{1}{2}$ then $-\delta \ge -\frac{1}{2}$

$2-\frac{1}{2} \le 2-\delta < x$

$\frac{3}{2} < x$ so $3 < 2x = |2x|$

and $\frac{1}{3} > \frac{1}{|2x|}$

Therefore

$$\left|\frac{1}{x} - \frac{1}{2}\right| = \left|\frac{1}{x}\cdot\frac{2}{2} - \frac{1}{2}\cdot\frac{x}{x}\right| = \frac{|2-x|}{|2x|} < \frac{\delta}{|2x|} < \frac{\delta}{3} \le \frac{3\varepsilon}{3} = \varepsilon$$

37 $\lim_{x\to 4} \sqrt{x} = 2$

Let $\varepsilon > 0$ be arbitrary and choose $\delta = \min(4, 2\varepsilon)$

Then $0 < |x-4| < \delta$ implies optional... add a condition on x to ensure $\sqrt{x}$ is not imaginary.

$-\delta < x-4 < \delta$ so $4-\delta < x < 4+\delta$ as long as δ is no bigger than 4, then $x > 0$.

Therefore

$$|\sqrt{x} - 2| = \left|(\sqrt{x}-2)\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}\right| = \frac{|x-4|}{|\sqrt{x}+2|} < \frac{\delta}{|\sqrt{x}+2|} \le \frac{\delta}{2} \le \frac{2\varepsilon}{2} = \varepsilon$$

Limit Laws Suppose that c is a constant and the limits

$$① \lim_{x \to a} f(x) = L_1 \quad \text{and} \quad ② \lim_{x \to a} g(x) = L_2$$

exist. Then

1. ③ $\lim_{x \to a} [f(x) + g(x)] = \overbrace{\lim_{x \to a} f(x)}^{L_1} + \overbrace{\lim_{x \to a} g(x)}^{L_2}$

Let $\varepsilon > 0$ be arbitrary.

Since $\lim_{x \to a} f(x) = L_1$ then for $\varepsilon_1 = \underline{\qquad\qquad}$

there is $\delta_1 > 0$ such that

$0 < |x - a| < \delta_1$ implies $|f(x) - L_1| < \varepsilon_1$

Same thing for $\lim_{x \to a} g(x) = L_2$ here.

Choose $\delta = \underline{\qquad\qquad}$

Then $0 < |x - a| < \delta$ implies

$|f(x) + g(x) - (L_1 + L_2)| < \quad \cdot \quad \cdot \quad - \quad - \quad - \qquad \varepsilon$

Finish on Monday...