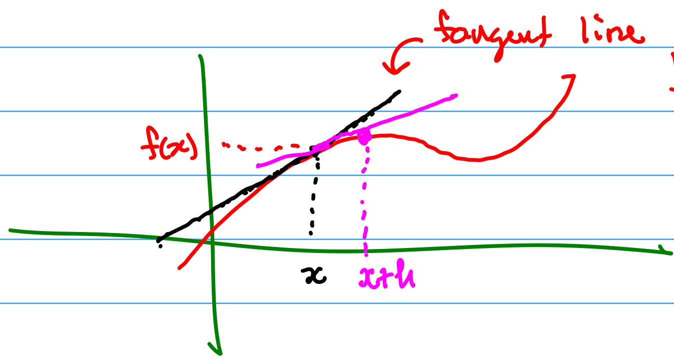
Revisit the derivative with more experiece with limits



need to know the slope.

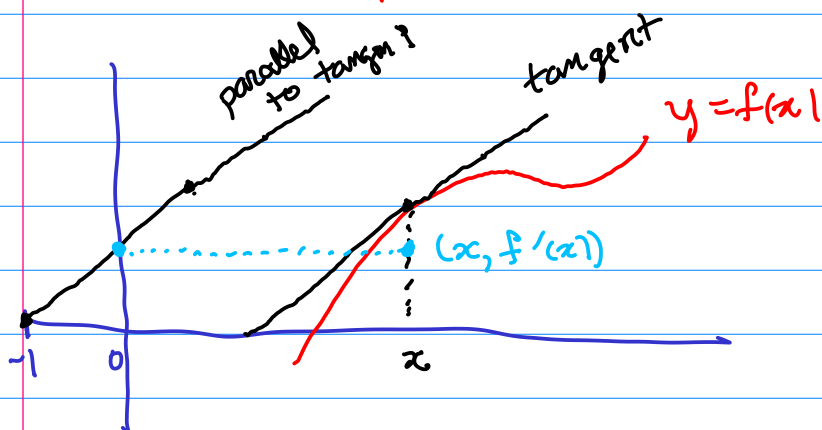
slope of the secant line through $(x, f(x))$ and $(x+h, f(x+h))$ approximates the slope of the tangent better and better as h gets smaller...

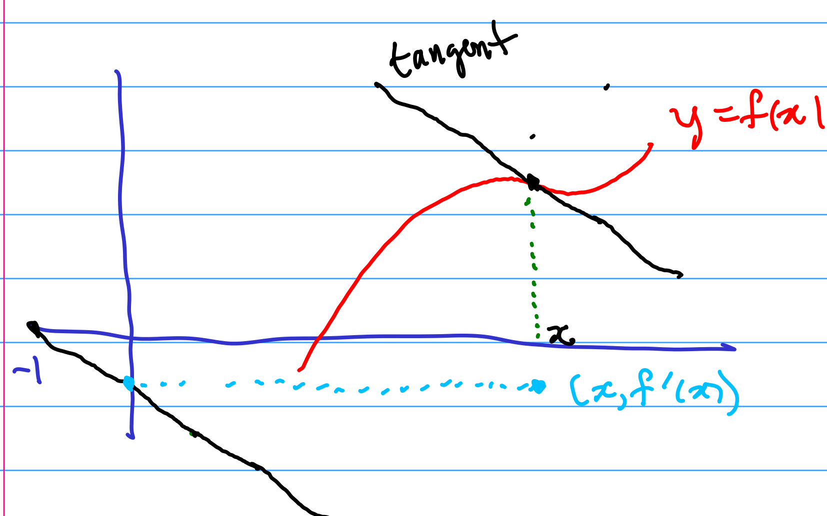
In symbols

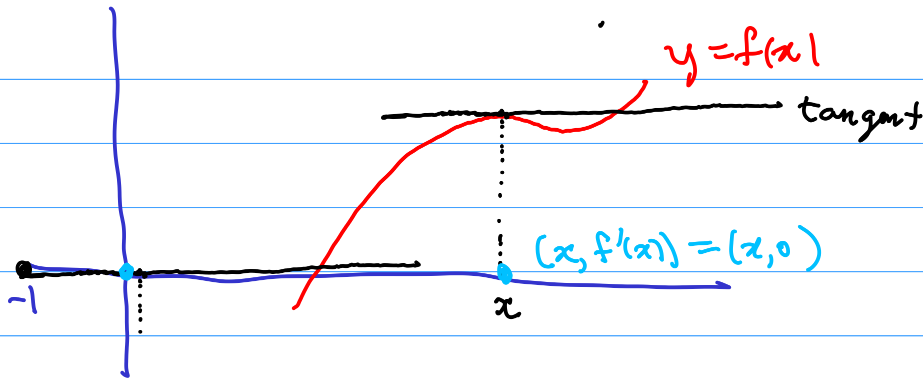
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h-x} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

slope of tangent

approximating parameter







applications of derivatives to finding maxima and minima of functions

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

Same except changed the parameter used to control the approx.

let $\underbrace{a = x + h}_{h = a - x}$ then $a \to x$ as $h \to 0$

$x = \lim_{h\to 0} x + h$ = $a = x$

$$\boxed{f'(x) = \lim_{a\to x} \frac{f(a) - f(x)}{a - x}}$$

Example $f(x) = \frac{1}{\sqrt{x}}$ then

$$f'(x) = \lim_{a\to x} \frac{\frac{1}{\sqrt a} - \frac{1}{\sqrt x}}{a - x} = \lim_{a\to x} \frac{-1}{(\sqrt a)(\sqrt x)(\sqrt x + \sqrt a)} = \frac{-1}{\sqrt x\sqrt x\,(\sqrt x + \sqrt x)} = \frac{-1}{2(\sqrt x)^3}$$

algebraic simplifications

$$\frac{\frac{1}{\sqrt a} - \frac{1}{\sqrt x}}{a - x} = \frac{1}{a-x}\left(\frac{1}{\sqrt a}\frac{\sqrt x}{\sqrt x} - \frac{1}{\sqrt x}\frac{\sqrt a}{\sqrt a}\right)$$

$$= \frac{1}{a-x}\left(\frac{\sqrt x - \sqrt a}{\sqrt a\sqrt x}\right)\left(\frac{\sqrt x + \sqrt a}{\sqrt x + \sqrt a}\right) = \frac{\overset{-1}{x-a}}{(a-x)(\sqrt a)(\sqrt x)(\sqrt x + \sqrt a)}$$

$$= \frac{-1}{(\sqrt a)(\sqrt x)(\sqrt x + \sqrt a)}$$

In summary.

If $f(x)=\frac{1}{\sqrt{x}}$ then $f'(x)=\frac{-1}{2(\sqrt{x})^3}=-\frac{1}{2}x^{-3/2}$

Quiz on Tuesday...

- Homework 5: Section 2.5#35,37,39,40,53,55,56

35–38 Use continuity to evaluate the limit.

§2.5#35 $\lim_{x\to 2} x\sqrt{20-x^2}$

- Since $20-x^2$ is a polynomial then $20-x^2$ is continuous
- Since $\lim_{x\to 2} 20-x^2 = 20-4=16>0$ and $\sqrt{\ }$ is continuous for positive numbers then the composition $\sqrt{20-x^2}$ is continuous
- Since x and $\sqrt{20-x^2}$ are continuous the so is their product.

Therefore $\lim_{x\to 2} x\sqrt{20-x^2} = 2\sqrt{20-2^2}=2\sqrt{16}=8$.

§2.5#39

39–40 Show that f is continuous on $(-\infty, \infty)$.

39. $f(x)=\begin{cases} 1-x^2 & \text{if } x\leqslant 1 \\ \ln x & \text{if } x>1 \end{cases}$

Case $x<1$ then $f(x)$ is continuous because $1-x^2$ is a polynomial

Case $x>1$ then x is positive and the function $\ln x$ is continuous.

Case $x=1$. Need to check the left and right limits agree and are equal to $f(1)=1-1^2=0$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(1-x^2) = 1-1^2 = 0 = f(1)$ ☑

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \ln x = \ln 1 = 0 = f(1)$ ☑

Since left and right agree with $f(1)$ then $\lim_{x\to 1} f(x) = f(1)$ so cont. here too!

53–56 Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

53. $x^4 + x - 3 = 0, \quad (1, 2)$

54. $\ln x = x - \sqrt{x}, \quad (2, 3)$

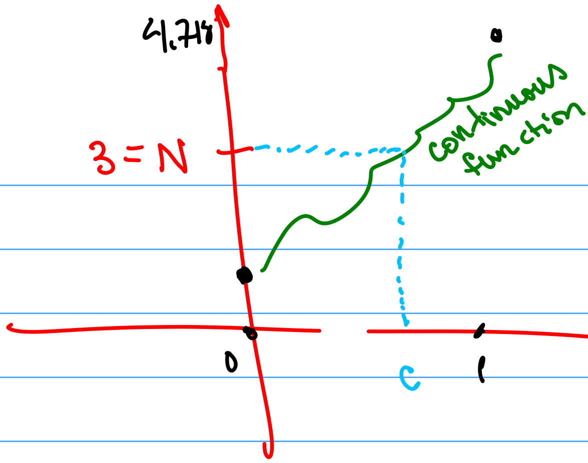
55. $e^x = 3 - 2x, \quad (0, 1)$

Let $f(x) = \underbrace{e^x + 2x}$ (trying to solve $f(x)=3$)

Since $f(x)$ is continuous and

$f(0) = e^0 + 2\cdot 0 = 1$

$f(1) = e^1 + 2\cdot 1 \approx 2.718 + 2 \approx 4.718$



Since

$f(0) < 3 < f(1)$

the intermediate value theorem says there is $c \in (0,1)$ such that $f(c) = 3$.