Homework 6: Section 2.6#15,16,19,27,31,34,35

15. $\lim_{x\to\infty} \frac{3x-2}{2x+1}$ **16.** $\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1}$ **19.** $\lim_{t\to\infty} \frac{\sqrt{t}+t^2}{2t-t^2}$ **34.** $\lim_{x\to-\infty} \frac{1+x^6}{x^4+1}$

27. $\lim_{x\to\infty} (\sqrt{9x^2+x}-3x)$ **31.** $\lim_{x\to\infty} \frac{x^4-3x^2+x}{x^3-x+2}$ **35.** $\lim_{x\to\infty} \arctan(e^x)$

Short cut related to orders of infinity

$\alpha > 1/2$ $\quad \beta > \alpha$ $\quad b > 2$

$$\log x \ll \sqrt{x} \ll x^\alpha \ll x^\beta \ll 2^x \ll b^x \ll x^x$$

Little more general

$0 < \alpha < \beta$ $\quad 1 < a < b$

$$\log x \ll x^\alpha \ll x^\beta \ll a^x \ll b^x \ll x^x$$

15. $\lim_{x\to\infty} \frac{3x-2}{2x+1} = \lim_{x\to\infty} \frac{3\cancel{x}}{2\cancel{x}} = \frac{3}{2}$

since both numerator and denominator tend to infinity, keep the term in the numerator that goes fastest and the one in the denominator that goes fastest

16. $\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} = \lim_{x\to\infty} \frac{-x^2}{x^3} = \lim_{x\to\infty} \frac{-1}{x} = 0$

19. $\lim_{t\to\infty} \frac{\underbrace{\sqrt{t}}_{t^{1/2}}+t^2}{2t-t^2} = \lim_{t\to\infty} \frac{t^2}{-t^2} = -1$

27. $\lim_{x\to\infty} (\sqrt{9x^2+x}-3x)$

different ... back to simplify and then use limit laws.

simplify by creating a difference of squares

$$(\sqrt{9x^2+x}-3x)\frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x} = \frac{9x^2+x-(3x)^2}{\sqrt{9x^2+x}+3x}$$

$$\lim_{x\to\infty} (\sqrt{9x^2+x}-3x) = \lim_{x\to\infty} \frac{x}{\sqrt{9x^2+x}+3x} = \lim_{x\to\infty} \frac{x}{\sqrt{9x^2}+3x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{9x^2}+3x} = \lim_{x\to\infty} \frac{x}{3x+3x} = \frac{1}{6}$$

since $x>0$

34. $\displaystyle\lim_{x\to-\infty}\frac{1+x^6}{x^4+1} = \lim_{y\to\infty}\frac{1+(-y)^6}{(-y)^4+1} = \lim_{y\to\infty}\frac{1+y^6}{y^4+1} = \lim_{y\to\infty}\frac{y^6}{y^4} = \lim_{y\to\infty} y^2 = \infty$

$y=-x$ $\quad y\to\infty$ as $x\to-\infty$

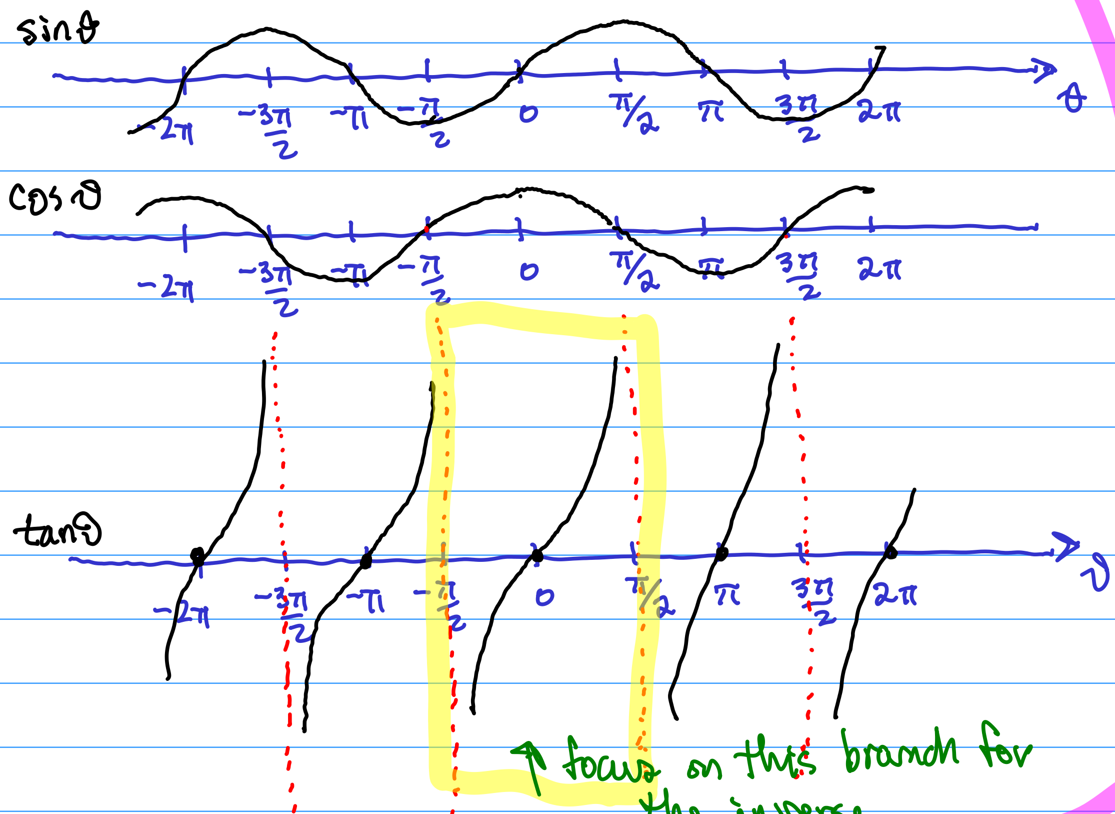
31. $\displaystyle\lim_{x\to\infty}\frac{x^4-3x^2+x}{x^3-x+2} = \lim_{x\to\infty}\frac{x^4}{x^3} = \infty$

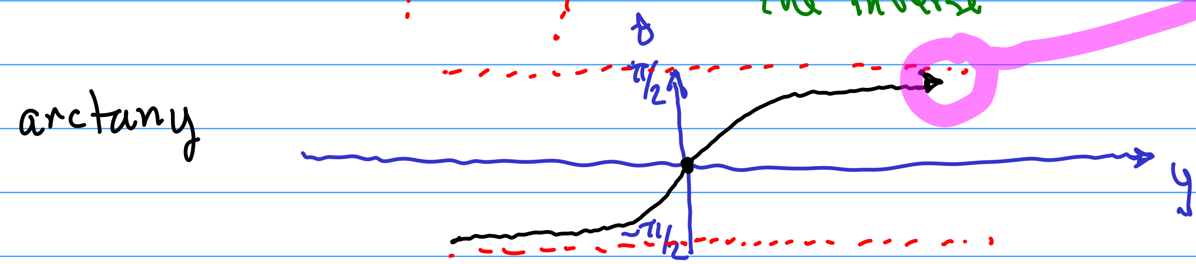
35. $\displaystyle\lim_{x\to\infty}\arctan(e^x) = \lim_{y\to\infty}\arctan y = \pi/2$

$y=e^x$ then $y\to\infty$ when $x\to\infty$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

↑ continuous except where $\cos\theta=0$ since sine and cosine are continuous and tangent is a quotient of continuous functions.





Rules for derivatives.

If $f(x) = \frac{1}{\sqrt{x}}$ then $f'(x) = \frac{-1}{2(\sqrt{x})^3} = -\frac{1}{2} x^{-3/2}$

If $f(x) = x$ then $f'(x) = 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}} = 1.$$

If $f(x) = cx$ then $f'(x) = c\,g'(x) = c \cdot 1 = c$

$g(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c(x+h) - cx}{h} = \lim_{h \to 0} \frac{\cancel{cx} + c\cancel{h} - \cancel{cx}}{\cancel{h}} = c$$

$$= \lim_{h \to 0} \frac{c(\cancel{x} + \cancel{h} - \cancel{x})}{\cancel{h}} = c$$

$$\boxed{\text{If } f(x) = c\,g(x) \text{ then } f'(x) = c\,g'(x)}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c\,g(x+h) - c\,g(x)}{h} = \lim_{h \to 0} \frac{c(g(x+h) - g(x))}{h}$$

$$= c \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = c\,g'(x)$$

$$\boxed{\text{If } f(x) = x\,g(x) \text{ then } f'(x) = x\,g'(x) + g(x)}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)g(x+h) - x\,g(x)}{h}$$

$$= \lim_{h \to 0} \frac{x g(x+h) + h g(x+h) - x g(x)}{h}$$

$$= \lim_{h \to 0} \frac{x(g(x+h) - g(x)) + h g(x+h)}{h}$$

$$= \lim_{h \to 0} \left(\frac{x(g(x+h) - g(x))}{h} + \frac{h g(x+h)}{h} \right)$$

$$= \lim_{h \to 0} \frac{x(g(x+h) - g(x))}{h} + \lim_{h \to 0} \frac{\cancel{h} g(x+h)}{\cancel{h}},$$

$$= x \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} + \lim_{h \to 0} g(x+h)$$

assuming g is cont.

$$= x g'(x) + g(x)$$