If $f(x) = \frac{1}{\sqrt{x}}$ then $f'(x) = \frac{-1}{2(\sqrt{x})^3} = -\frac{1}{2}x^{-3/2}$

If $f(x) = x$ then $f'(x) = 1$

If $f(x) = cx$ $g(x)$ then $f'(x) = c\,g'(x) = c \cdot 1 = c$

If $f(x) = c\,g(x)$ then $f'(x) = c\,g'(x)$

If $f(x) = x\,g(x)$ then $f'(x) = x\,g'(x) + g(x)$

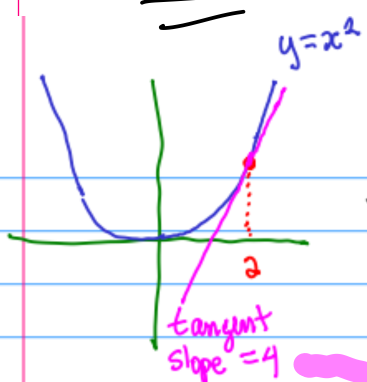
If $f(x) = x$ then $f'(x) = 1$

$g(x) = x$ then $g'(x) = 1$

If $f(x) = x\,g(x)$ then $f'(x) = x\,g'(x) + g(x)$

If $f(x) = x \cdot x = x^2$ then $f'(x) = x \cdot 1 + x = 2x$

From Lecture 2



$f(x) = x^2$

line tangent to $y = x^2$ at $x_0 = 2$

in terms of derivatives

$\underbrace{\text{slope of the line tangent to } y = x^2 \text{ at } x_0 = 2}$

called the derivative of $y = x^2$ at $x_0 = 2$

$f'(x) = x \cdot 1 + x = 2x$

$f'(2) = 2 \cdot 1 + 2 = 2 \cdot 2 = 4$

If $g(x) = x^2$ then $g'(x) = 2x$

→ If $f(x) = x\,g(x)$ then $f'(x) = x\,g'(x) + g(x)$

If $f(x) = x \cdot x^2 = x^3$ then $f'(x) = x \cdot 2x + x^2 = 3x^2$

the derivative of x^3 is $3x^2$

To connect to the first week... directly approximate the slope of the tangent by secant lines

definition of derivative in symbols

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

← simplifications to figure out what happens when $h \to 0$

$$\frac{(x+h)^3 - x^3}{h} = \frac{(x+h)\overbrace{\boxed{(x+h)^2}}^{g(x+h)} - x^3}{h} = \frac{x(x+h)^2 + h(x+h)^2 - x^3}{h}$$

$$= \frac{x(x^2 + 2xh + h^2) + h(x+h)^2 - x^3}{h}$$

$$= \frac{\cancel{x^3} + 2x^2h + xh^2 + h(x+h)^2 - \cancel{x^3}}{h}$$

$$= \frac{\cancel{h}(2x^2 + xh + (x+h)^2)}{\cancel{h}} = 2x^2 + xh + (x+h)^2$$

Polynomial is a continuous function

$$f'(x) = \lim_{h \to 0} \left[2x^2 + xh + (x+h)^2\right] = 2x^2 + 0 + (x+0)^2 = 3x^2$$

If $g(x) = x^3$ then $g'(x) = 3x^2$

If $f(x) = x\,g(x)$ then $f'(x) = x\,g'(x) + g(x)$

If $f(x) = x \cdot x^3 = x^4$ then $f'(x) = x \cdot 3x^2 + x^3 = (3+1)x^3 = 4x^3$ is 4-1

There is a pattern

If $f(x) = x^n$ then $f'(x) = n x^{n-1}$

works for any positive integer n that one can count up to using the rule

If $f(x) = x g(x)$ then $f'(x) = x g'(x) + g(x)$

Check: Suppose for some n I know

If $g(x) = x^n$ then $g'(x) = n x^{n-1}$

If $f(x) = x g(x)$ then $f'(x) = x g'(x) + g(x)$

If $f(x) = x \cdot x^n = x^{n+1}$ then $f'(x) = x \cdot n x^{n-1} + x^n = (n+1)x^n$

this is the same patter for $n+1$ which counts up by one. (Note: this is called induction).

Conclusion

If $f(x) = x^n$ then $f'(x) = n x^{n-1}$

works for any positive integer n that one can count to using the rule

Notation:

$$f'(x) = \lim_{h \to 0} \frac{\overbrace{f(x+h) - f(x)}^{\text{slopes of approximating secant lines}}}{h}$$

this is a fraction

Notation due to Lagrange.

slope of tangent

parameter to control the approximation

Leibnitz

$$\frac{d}{dx} f(x)$$

slope of tangent.

Newton notation

used $\cdot$ instead of $'$

still used in physics for time derivatives that represent velocities and acceleration

If $f(x) = \frac{1}{\sqrt{x}}$ then $f'(x) = \frac{-1}{2(\sqrt{x})^3} = \frac{-1}{2} x^{-3/2}$

If $f(x) = x$ then $f'(x) = 1$

If $f(x) = cx$ $\leftarrow g(x)$ then $f'(x) = c\, g'(x) = c \cdot 1 = c$

If $f(x) = c\,g(x)$ then $f'(x) = c\,g'(x)$

If $f(x) = x\,g(x)$ then $f'(x) = x\,g'(x) + g(x)$

If $f(x) = x^n$ then $f'(x) = n x^{n-1}$

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{-1}{2} x^{-3/2}$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(cx) = c.$$

$$\frac{d}{dx}\left(c\,g(x)\right) = c\,g'(x) = c\frac{d}{dx}g(x)$$

$$\frac{d}{dx}\left(x\,g(x)\right) = x\,g'(x) + g(x)$$
$$= x\frac{d}{dx}g(x) + g(x)$$

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

Slight modification

$$\frac{d x^n}{dx} = n x^{n-1}$$

think as a fraction

$$y = f(x)$$

$$\Delta x = a - x$$

$$\boxed{\frac{dy}{dx}} = \frac{d f(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{a \to x} \frac{f(a) - f(x)}{a - x} = \lim_{\Delta x \to 0} \boxed{\frac{\Delta y}{\Delta x}}$$

idea is that the Δ turns into d in the limit

Good notation is kept around because it's good...

Note we've already done $f(x)=\frac{1}{x}$ the first week...

more general

How about $f(x)=\dfrac{g(x)}{x}$ what is $f'(x)$?

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{g(x+h)}{x+h}-\frac{g(x)}{x}}{h}$$

Simplify

$$\frac{\frac{g(x+h)}{x+h}-\frac{g(x)}{x}}{h}=\frac{1}{h}\,\frac{g(x+h)x-g(x)(x+h)}{(x+h)x}$$

$$=\frac{1}{h}\,\frac{g(x+h)x-g(x)x-g(x)h}{(x+h)x}=\frac{1}{h}\,\frac{x\,\boxed{(g(x+h)-g(x))}}{(x+h)(x)}-\frac{g(x)\cancel{h}}{\cancel{h}(x+h)x}$$

$$=\frac{x}{(x+h)x}\,\frac{g(x+h)-g(x)}{h}-\frac{g(x)}{(x+h)x}$$

$$f'(x)=\lim_{h\to 0}\left(\frac{x}{(x+h)x}\boxed{\frac{g(x+h)-g(x)}{h}}-\frac{g(x)}{(x+h)x}\right)$$

derivative

$$=\lim_{h\to 0}\frac{x}{(x+h)x}\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}-\lim_{h\to 0}\frac{g(x)}{(x+h)x}$$

$$=\frac{1}{x}g'(x)-\frac{1}{x^2}g(x)$$

If $f(x)=\dfrac{g(x)}{x}$ then $f'(x)=\dfrac{1}{x}g'(x)-\dfrac{1}{x^2}g(x)$