Chain rule

If $f(x) = u(v(x))$ then $f'(x) = u'(v(x))\, v'(x)$

Finding derivative on $\ln x$

$y = e^x$

$\ln y = \ln e^x = x \ln e = x$

this means ln is the inverse function of exponential

also this

$x = e^{\ln x}$

$f(x) = \underbrace{e^{\ln x}}_{\text{composition of functions}} = u(v(x))$

$f(x) = x$

$u(x) = e^x$ $\quad u'(x) = e^x$

$v(x) = \ln x$ $\quad v'(x) = ?$

$f'(x) = u'(v(x))\, v'(x)$

$f'(x) = 1$

Therefore

$u'(v(x))\, v'(x) = 1$

$\boxed{e^{\ln x}}\, v'(x) = 1$

$x\, v'(x) = 1$

$v'(x) = \frac{1}{x}$

If $v(x) = \ln x$ then $v'(x) = \frac{1}{x}$

Different base logarithm

$x = 10^{\log_{10} x}$

basically definition that $\log_{10} x$ is the inverse of 10^x

$u(x) = 10^x$ $\quad u'(x) = 10^x \ln 10$

$v(x) = \log_{10} x$ $\quad v'(x) = ?$

Last time

$\frac{d}{dx} a^x = a^x \ln a$

$f(x) = 10^{\log_{10} x} = u(v(x))$

$f'(x) = u'(v(x))\,v'(x) = \left(10^{v(x)} \ln 10\right) v'(x) = \left(\underbrace{10^{\log_{10} x}}_{x} \ln 10\right) v'(x) = (x \ln 10)\, v'(x)$

same

$f(x) = 10^{\log_{10} x} = x$

$f'(x) = 1$

$(x \ln 10)\, v'(x) = 1$

$v'(x) = \dfrac{1}{x \ln 10}$

If $v(x) = \log_{10} x$ then $v'(x) = \dfrac{1}{x \ln 10}$

If $v(x) = \log_a x$ then $v'(x) = \dfrac{1}{x \ln a}$ $\quad a > 0$

One more application of chain rule

$f(x) = x^{\alpha}$ for any α not necessarily a positive integer...

If $f(x) = x^n$ then $f'(x) = n x^{n-1}$

works for any positive integer n that one can count to using the rule

$f(x) = \boxed{x^{\alpha}} = \left(e^{\ln x}\right)^{\alpha} = e^{\alpha \ln x} = u(v(w(x)))$

$u(x) = e^x \qquad u'(x) = e^x$

$v(x) = \alpha x \qquad v'(x) = \alpha$

$w(x) = \ln x \qquad w'(x) = \frac{1}{x}$

$f'(x) = u'(v(w(x)))\, v'(w(x))\, w'(x) = e^{v(w(x))} \alpha \frac{1}{x} = e^{\alpha \ln x} \alpha \cdot \frac{1}{x}$

$= x^{\alpha} \cdot \alpha \cdot \frac{1}{x} = \alpha x^{\alpha - 1}$ for any α.

This also works for $\alpha = 0$! But since I used $\ln x$ in the work then we need $x > 0$. to avoid complex number...

When $\alpha = n$ is a positive integer it doesn't matter if x is zero or negative because x^n just means multiply x by itself n times.

when $\alpha = -n$ is a negative integer then x^{-n} means multiply $\frac{1}{x}$ by itself n times. In this case x can be negative but not zero.

when $\alpha = \frac{1}{2}$ then $x^{\frac{1}{2}} = \sqrt{x}$ and we did this using limits (on the quiz yesterday $\sqrt{x-9}$). At anyrate we'll assume $x > 0$ to avoid complex and imaginary numbers

If $f(x) = x^{\alpha}$ then $f'(x) = \alpha x^{\alpha - 1}$ for any α and $x > 0$

if $f(x) = u(x)v(x)$ then $f'(x) = u(x)\,v'(x) + u'(x)\,v(x)$.
$f'(x) = u'(x)\,v(x) + u(x)\,v'(x)$

Use product rule to find the derivative of a quotient.

In Leibnitz notation

$$\frac{d}{dx}\left(u(x)v(x)\right) = u'(x).v(x) + u(x)\,v'(x) \quad \text{and} \quad \frac{d}{dx}\left(\frac{1}{g(x)}\right) = \frac{-g'(x)}{(g(x))^2}$$

minus sign came from here

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{d}{dx}\left(f(x)\cdot\frac{1}{g(x)}\right) = \frac{d}{dx}\left[u(x)v(x)\right] = u'(x)\,v(x) + u(x)\,v'(x)$$

$u(x) = f(x) \quad v(x) = \frac{1}{g}$

$$= f'(x)\frac{1}{g(x)} + f(x)\left(\frac{-g'(x)}{(g(x))^2}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Therefore

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Comes in front of the term with $g'(x)$ because g was in the denominator

This is the derivative of what was in the denominator and that's where the minus sign goes...

Derivatives of sine and cosine

remember trigonometry: Angle addition formula.

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

the term that is positive can be remembered by checking $a=0$, $b=0$ to make sure it works.