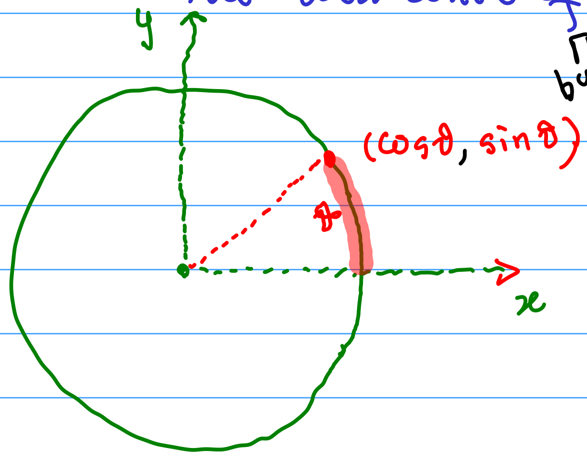
From last time

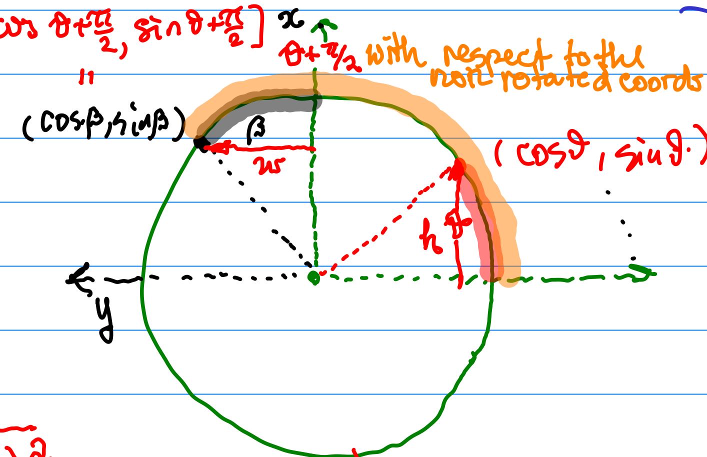
If $f(x) = \sin x$ then $f'(x) = \cos x$

Leibnitz notation $\frac{d}{dx} \sin x = \cos x$

What is the derivative of $\cos x$?



rotate by $\pi/2$

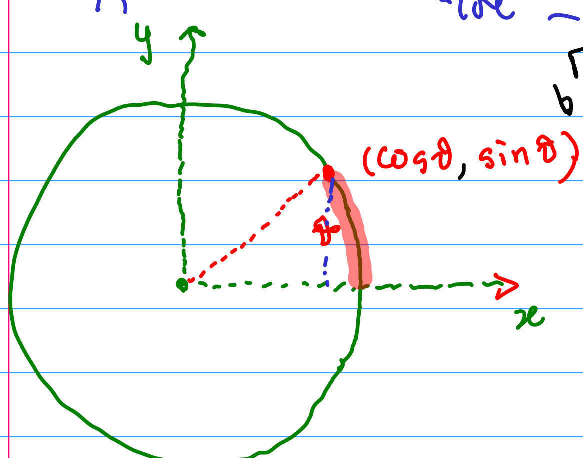


since w is in the 2nd quadrant then $x<0$ $\longrightarrow$ $w = \sqrt{\cos(\theta+\frac{\pi}{2})^2} = |\cos(\theta+\frac{\pi}{2})| = -\cos(\theta+\frac{\pi}{2})$

$h = \sin\theta$

Therefore $\boxed{\sin\theta = -\cos(\theta + \pi/2)}$

- is tive -



shift right by $\frac{\pi}{2}$ to get sin.

$\cos(\theta - \frac{\pi}{2}) = \sin\theta$

cosine is even $\cos(\frac{\pi}{2} - \theta) = \sin\theta$

let $\beta = -\theta$ $\cos(\frac{\pi}{2} + \beta) = \sin(-\beta)$

sine is odd then $\boxed{\cos(\frac{\pi}{2} + \beta) = -\sin\beta}$

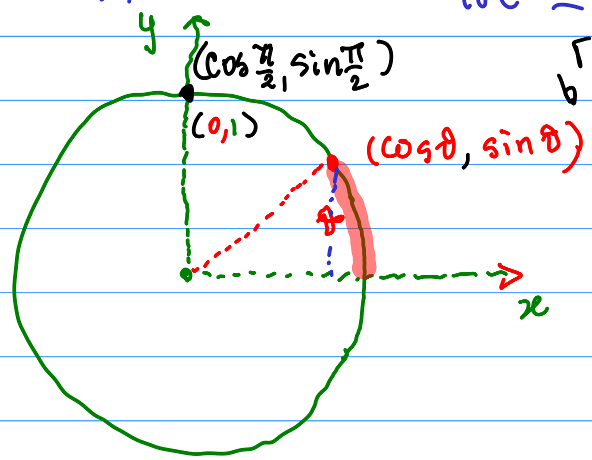
The same as $\boxed{\sin\theta = -\cos(\theta + \pi/2)}$

The Same

$$\sin(a+b) = \sin a\cos b + \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(\frac{\pi}{2} + \beta) = \underbrace{\cos\frac{\pi}{2}}_{0}\cos\beta - \underbrace{\sin\frac{\pi}{2}}_{1}\sin\beta = -\sin\beta$$



What is the derivation of $\cos x$?

$\cos(\underbrace{\theta - \frac{\pi}{2}}) = \sin\theta$

$x = \theta - \frac{\pi}{2}$

$\theta = x + \frac{\pi}{2}$

Thus $\cos x = \sin(x + \frac{\pi}{2})$

by chain rule

$$\frac{d}{dx}\cos x = u'(v(x))\,v'(x)$$

$$= \cos(x + \frac{\pi}{2}) \cdot 1$$

$\cos x = u(v(x))$

$u(x) = \sin x$

$v(x) = x + \frac{\pi}{2}$

$v'(x) = 1$

Therefore $\frac{d}{dx}\cos x = \cos\left(x+\frac{\pi}{2}\right) = -\sin x$

from before $\frac{d}{dx}\sin x = \cos x$

recall

$\cos\left(\frac{\pi}{2}+\beta\right) = -\sin\beta$

Angle addition

$\sin(a+b) = \sin a\cos b + \cos a\sin b$

$\cos(a+b) = \cos a\cos b - \sin a\sin b$

$\sin(x+b) = \sin x\cos b + \cos x\sin b$

$\frac{d}{dx}\sin(x+b) = \cos(x+b)\frac{d}{dx}(x+b) = \cos(x+b)$

$\frac{d}{dx}\left(\sin x\underbrace{\cos b}_{\text{const}} + \cos x\underbrace{\sin b}_{\text{const}}\right) = \cos x\cos b + (-\sin x)\sin b$

$= \cos x\cos b - \sin x\sin b$

Therefore

$\cos(x+b) = \cos x\cos b - \sin x\sin b$

which is the angle addition formula for cosine.

What about $\arcsin x$? What's the derivative?

$f(x) = \sin(\arcsin x)$ $\qquad f(x) = x$

$u(x) = \sin x \qquad u'(x) = \cos x \qquad f'(x) = 1$

$v(x) = \arcsin x$

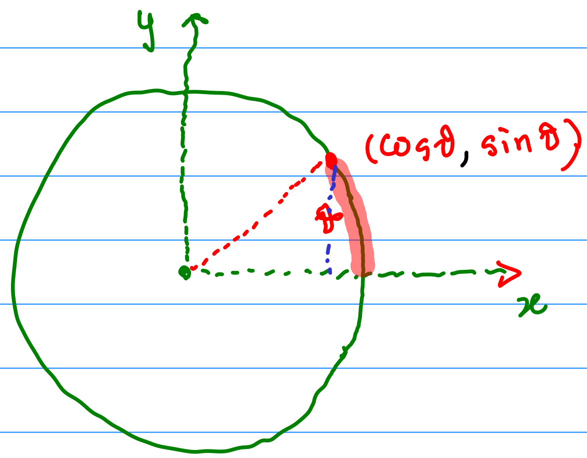
$f'(x) = u'(v(x))\,v'(x) = \cos(\arcsin x)\,v'(x)$

Therefore

$$\cos(\arcsin x)\, V'(x) = 1$$

So

$$V'(x) = \frac{1}{\cos(\underbrace{\arcsin x}_{\theta})}$$



Since $(\cos\theta, \sin\theta)$ are on the unit circle then

$$\cos^2\theta + \sin^2\theta = 1$$

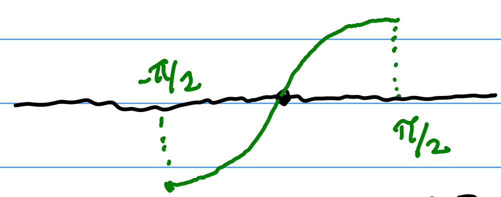
$$\cos^2\theta = 1 - \sin^2\theta$$

$$\boxed{\underbrace{\cos\theta}_{} = \pm\sqrt{1-\sin^2\theta}}$$

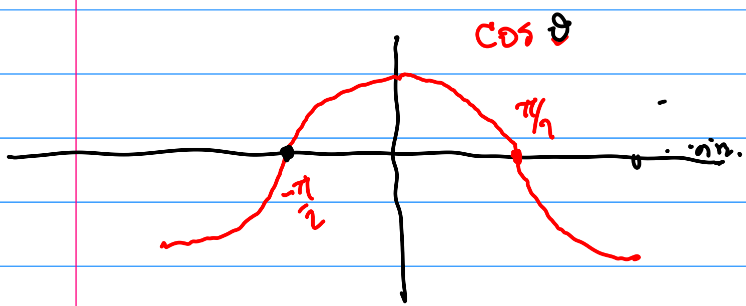
which one? depends on θ

Note $\sin\theta$ is not invertible — doesn't pass horizontal line test.

Thus restriction of $\sin\theta$ passes the horizontal line test in



$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$



notice that $\cos\theta \geq 0$ when $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

Thus $+$ is right

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$V'(x) = \frac{1}{\cos(\underbrace{\arcsin x}_{\theta})} = \frac{1}{\sqrt{1-(\sin(\arcsin x))^2}} = \frac{1}{\sqrt{1-x^2}}$$

Leibnitz notation

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}} \quad \text{for} \quad x \in (-1, 1).$$

Thursday is a quiz...

More derivative rules Friday!