What is the derivative of $\tan x$?

$f(x) = \dfrac{\sin x}{\cos x}$ so want to use quotient rule and the derivatives

$\dfrac{d}{dx}\sin x = \boxed{\cos x}$ $\qquad \dfrac{d}{dx}\cos x = \boxed{-\sin x}$

$$f'(x) = \frac{\left(\frac{d}{dx}\sin x\right)\cos x - \sin x\left(\frac{d}{dx}\cos x\right)}{(\cos x)^2}$$

minus sign goes with the derivative of the denominator.

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \boxed{\frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}}$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

Remark, only for trigonometric functions

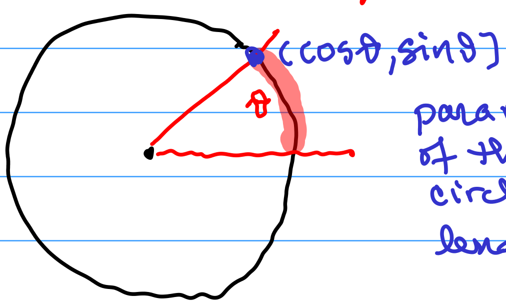
$\sin^2 x = (\sin x)^2$

$\cos^2 x = (\cos x)^2$

short-hand notation.

$$= \frac{\overbrace{\cos^2 x + \sin^2 x}^{\text{Pythagorean theorem}}}{\cos^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

Recall



definition of sine and cosine.

parameterization of the unit circle by arc length...

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$

What is the derivative of $\arctan x$?

$\cos^2 x + \sin^2 x = 1$

$\dfrac{\cos^2 x}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} \longrightarrow \quad 1 + \tan^2 x = \sec^2 x$

inverse of tan

$f(x) = \tan(\arctan x)$ then $f(x) = x$

chain rule ...

$f'(x) = 1$

Oct 06 -- 3.5 Implicit Differentiation ← still need to do implicit differentiation..
Quiz 9 (recitation)
Webassign HW 3.2, 3.3
Oct 08 -- 3.6 Derivatives of Log and Inverse Trig Functions
Quiz 10 (recitation)
Oct 10 -- 3.9 Related Rates

$u(x) = \tan x \quad u'(x) = \sec^2 x$

$v(x) = \arctan x \quad v'(x) = ?$

$f'(x) = u'(v(x))\, v'(x)$

$= \sec^2(\arctan x)\, v'(x) = 1$

Therefore

$$\boxed{v'(x) = \frac{1}{\sec^2(\arctan x)}}$$

θ

Now simplify...

$\boxed{1 + \tan^2\theta = \sec^2\theta}$

Thus

$$v'(x) = \frac{1}{1 + \tan^2(\arctan x)}$$

$$= \frac{1}{1 + [\tan(\arctan x)]^2} = \frac{1}{1 + x^2}$$

Leibnitz notation $\quad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.

$y = \arctan x$

means $x = \tan y$

$$\frac{d \arctan x}{dx} = \frac{dy}{dx}$$

implicit differentiation allows to find dy/dx using the chain rule.

think of $y = y(x)$ differentiate both sides of equation

$x = \tan y$

$$\frac{dx}{dx} = \frac{d}{dx}\tan y$$

chain rule

$$1 = (\sec^2 y)\frac{dy}{dx}$$

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

end with implicit differentiation

optional last step

Now express y in terms of x

$$\boxed{\frac{d}{dx}\arctan x = \frac{1}{\sec^2 \arctan x}}$$

same

$\arcsin x = \sin^{-1} x$ emphasize this is inverse of sine.

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}} \longrightarrow \boxed{\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}}$$

Summary of derivative formula...

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$\frac{d}{dx} \sin x =$ $\qquad$ $\frac{d}{dx} \cos x$

all the formula for exponential and logarithm...

quotient, product and chain rules...