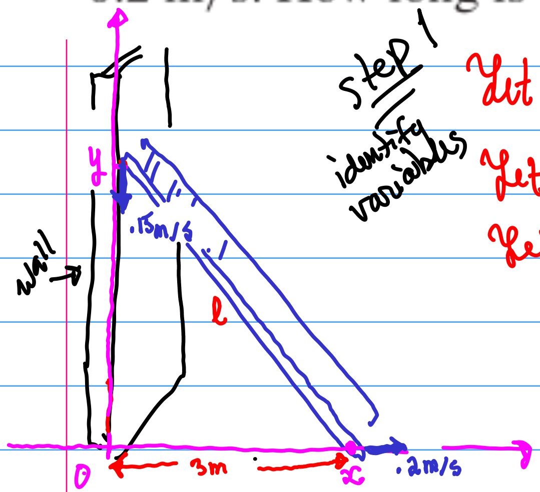
33. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?

negative



step 1 identify variables

Let l be the length of the ladder...

Let y be the height of the top of the ladder

Let x be the position of the base of the ladder.

step 2 equations between variables

Pythagorean theorem

$$x^2 + y^2 = l^2$$

Step 3 data in the problem

$$\frac{dy}{dt} = -0.15 \text{ m/s.} \qquad x = 3\text{m} \qquad \frac{dx}{dt} = 0.2\text{m/s}$$

Step 4 implicit differentiation

Think of $x = x(t)$ and $y = y(t)$. Note l is constant.

$$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt} l^2$$

chain rule

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Solve for unknowns...

$$2(3)(0.2) + 2y(-0.15) = 0$$

$$y = \frac{(3)(0.2)}{0.15} = \frac{(3)(2)}{1.5} = \frac{6}{3/2} = 6\,\frac{2}{3} = 4$$

Answer the question

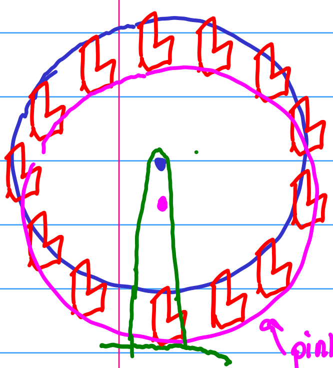
How long is the ladder? $\boxed{5 \text{ m}}$ answer needs dimensional units to be fully correct.

since $x^2 + y^2 = l^2$ so $l = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ m

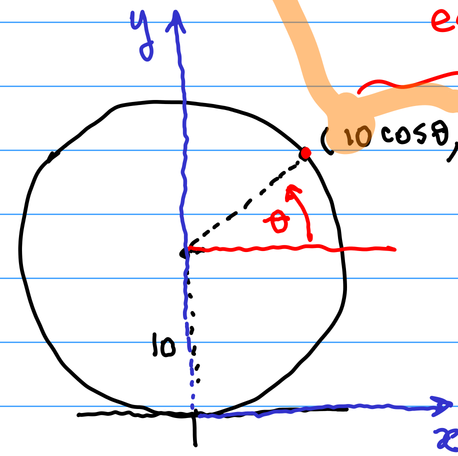
46. A Ferris wheel with a radius of 10 m is rotating at a rate of one revolution every 2 minutes. How fast is a rider rising when his seat is 16 m above ground level?

step 1

simple picture



pink circle is the same as the blue one



equation giving position

$\big(10\cos\theta,\ 10 + 10\sin\theta\big)$

$$\begin{cases} x = 10\cos\theta \\ y = 10 + 10\sin\theta \end{cases}$$

step 3

step 2 Let y be the height of the rider above ground
x be the displacement from the support
θ be the angle in the picture giving the position of the wheel.

There are 2π radians in a complete rotation so

$$\frac{d\theta}{dt} = \frac{2\pi}{2}\ \frac{\text{radians}}{\text{minute}}$$

θ is measured in radians
t is measured in minutes

so $\dfrac{d\theta}{dt}$ is measured in radians/minute

$y = 16$ meters

shorthand

$[\theta] = \text{rad}$

$[t] = \text{min}$

$\left[\dfrac{d\theta}{dt}\right] = \dfrac{\text{rad}}{\text{min}}$

the brackets mean just what are the units of measurement.

If dealing with meters in an application, we like to avoid measuring time in minutes — both start with m. So let's use seconds for time.

$$\frac{d\theta}{dt} = \frac{\cancel{2}\pi}{\cancel{2}}\ \frac{\text{radians}}{\text{minute}} \cdot \boxed{\frac{1 \text{ minute}}{60 \text{ seconds}}} = \frac{\pi}{60}\ \text{rad/sec}$$

multiply by 1 while changing the units of measurement

$y = 16$ meters

$x = 10\cos\theta$

$\boxed{y = 10 + 10\sin\theta}$

step 3

Implicit differentiation:

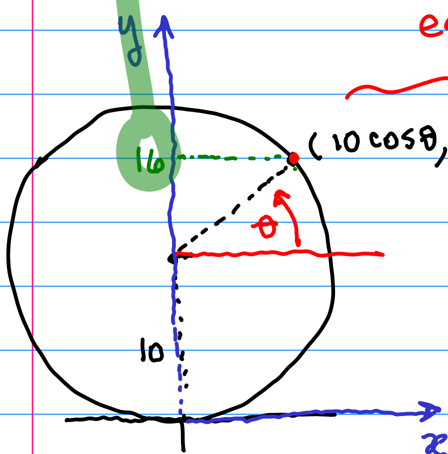
$$\frac{d}{dt} y = \frac{d}{dt}(10 + 10\sin\theta)$$

what is θ?

actually only need to know $\cos\theta$

$$\frac{dy}{dt} = 10 \frac{d}{dt}\sin\theta = 10\cos\theta \frac{d\theta}{dt}$$

equation giving position



$(10\cos\theta, 10 + 10\sin\theta)$

$$\begin{cases} x = 10\cos\theta \\ y = 10 + 10\sin\theta \end{cases}$$

step 3

$16 = 10 + 10\sin\theta$

$6 = 10\sin\theta$

$\sin\theta = \frac{6}{10} = \frac{3}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$

$$\cos\theta = \sqrt{1 - 9/25} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

clue we're on the right track if the numbers work out like magic...

$$\frac{dy}{dt} = \cancel{10}^{2} \cdot \frac{4}{\cancel{5}} \cdot \frac{\pi}{60} = \frac{2\pi}{15}\ \text{m/sec.}$$