where the max or min is

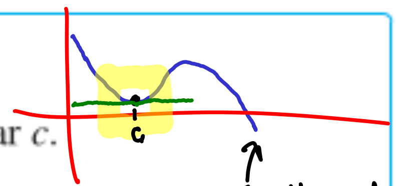
max or min of f

1 Definition Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute maximum** value of f on D if $f(c) \geqslant f(x)$ for all x in D.
- **absolute minimum** value of f on D if $f(c) \leqslant f(x)$ for all x in D.

2 Definition The number $f(c)$ is a

- **local maximum** value of f if $f(c) \geqslant f(x)$ when x is near c.
- **local minimum** value of f if $f(c) \leqslant f(x)$ when x is near c.



smaller values for f over here, but those are far from c.

3 The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

This is statement how the real number fill up the entire line with no gaps between them. So the point corresponding to the maximum is really there.

4 Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

at the top of a hill the ground is level ... same for the bottom of a valley..

6 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

rephrases Fermat's theorem...

7 If f has a local maximum or minimum at c, then c is a critical number of f.

algorithm to find max and minimums...

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

47. $f(x) = 12 + 4x - x^2, \quad [0, 5]$

endpoints

step **1.** Find the values of f at the critical numbers of f in (a, b).

> **6 Definition** A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

$$f'(x) = \frac{d}{dx}\left(12 + 4x - x^2\right) = 0 + 4 - 2x = 2(2-x)$$

defined everywhere so derivative exists..

Solve for x in $[0,5]$ where $2(2-x) = 0$.

$x = 2$ is a critical number.

$$f(2) = 12 + 4\cdot 2 - 2^2 = 12 + 8 - 4 = 16$$ largest

step **2.** Find the values of f at the endpoints of the interval.

$$f(0) = 12 + 4\cdot 0 - 0^2 = 12$$

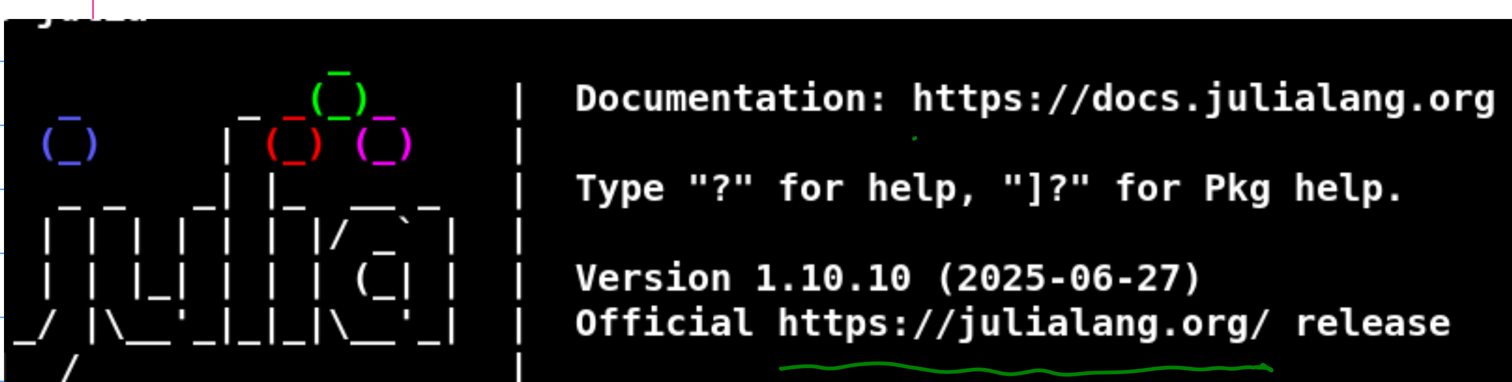
$$f(5) = 12 + 4\cdot 5 - 5^2 = 32 - 25 = 7$$ smallest

step **3.** The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

the absolute maximum of f on $[0,5]$ is $f(2) = 16$.

the absolute minimum of f on $[0,5]$ is $f(5) = 7$.

☑



look for this on Nevada day...

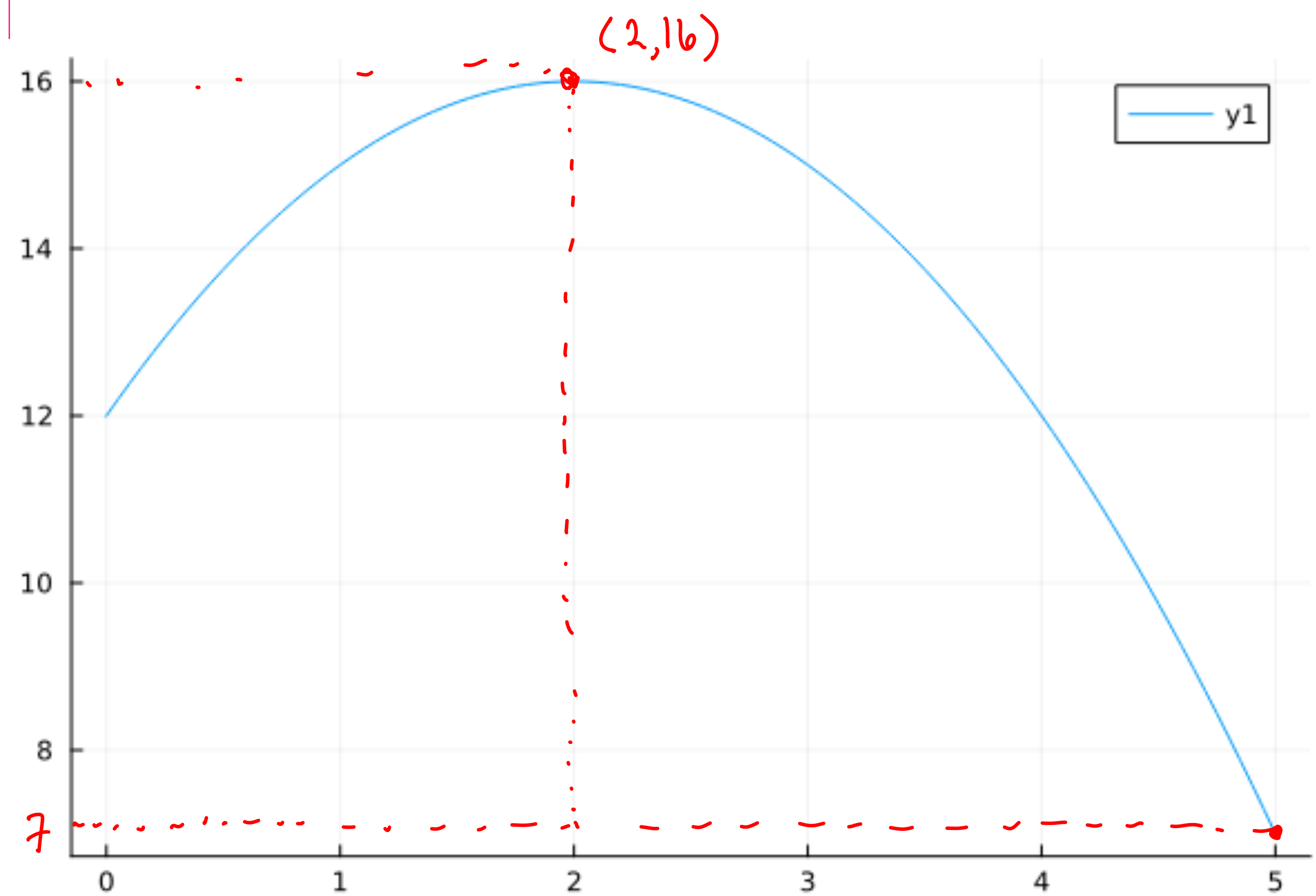
```
julia> using Plots

julia> f(x)=12+4*x-x^2
f (generic function with 1 method)

julia> plot(f,0:0.05:5)
```

The absolute maximum of f on [0,5] is f(2)=16. ☑

The absolute minimum of f on [0,5] is f(5)=7. ☑



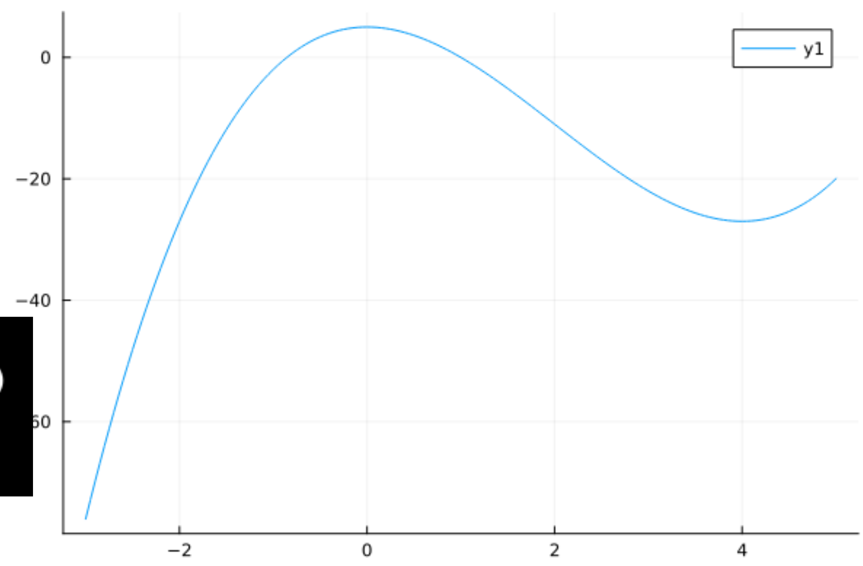
50. $f(x) = x^3 - 6x^2 + 5, \quad [-3, 5]$

endpoints

$f(x) = (x-6)x^2 + 5$

```
julia> f(x)=x^3-6*x^2+5
f (generic function with 1 method)

julia> plot(f,-3:0.05:5)
```



step 1 $f'(x) = 3x^2 - 12x = 3x(x-4) = 0$

critical numbers $x=0$ and $x=4$

$f(0) = 0^3 - 6\cdot 0^2 + 5 = 5$ largest of all choices

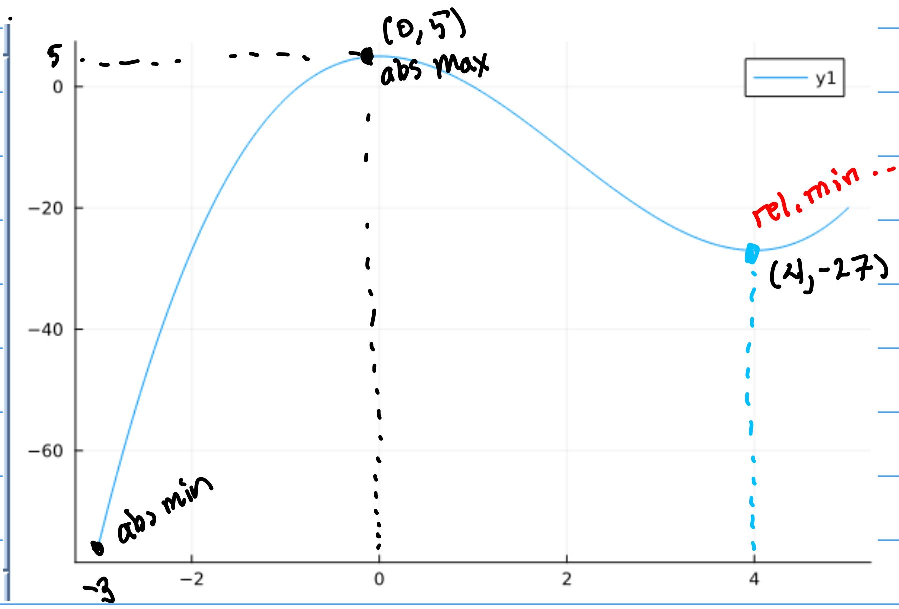
$f(4) = (-2)16 + 5 = -32+5 = -27$

step 2 $f(-3) = (-9)9 + 5 = -81+5 = -76$ smallest of all choices.

$f(5) = (-1)25 + 5 = -25+5 = -20$

step 3 The absolute maximum of f on $[-3, 5]$ is $f(0) = 5$. ☑

The absolute minimum of f on $[-3, 5]$ is $f(-3) = -76$. ☑



58. $f(t) = t + \cot(t/2), \quad [\pi/4, 7\pi/4]$

$\sin(a+b) = \sin a \cos b + \cos b \sin a$

$\cos(a+b) = \cos a \cos b - \sin b \sin a$

$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = 1 - \cos^2 x$

let $a = x$ and $b = x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$

solve for $\cos x$

$2\cos^2 x = 1 + \cos 2x$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\cos x = \frac{1}{\sqrt{2}}\sqrt{1 + \cos 2x}$

Change variables $2x = \theta$ then $x = \theta/2$

$\cos \theta/2 = \frac{1}{\sqrt{2}}\sqrt{1 + \cos\theta}$.

I know $\cos \frac{\pi}{2} = 1$. Therefore

$\theta = \frac{\pi}{2}$: $\cos \pi/4 = \frac{1}{\sqrt{2}}\sqrt{1 + \cos \pi/2} = \frac{2}{\sqrt{2}}$

$\theta = \pi/4$: $\cos \pi/8 = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{2}{\sqrt{2}}}$

$\vdots$