Exam on Friday!

Look for sample exams Tuesday...

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Oct 20 -- 4.2 The Mean Value Theorem
          Webassign HW 3.5, 3.6, 3.9
          Quiz 13 (recitation)
Oct 22 -- Review
Oct 24 -- Midterm
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Goal: To show if $f'(x) = g'(x)$ for all $x \in (a,b)$ then there is some constant c such that $f(x) = g(x) + C$ for all $x \in (a,b)$.

Fermat's theorem:

so the point corresponding to the maximum is really there.

4 Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

at the top of a hill the ground is level ... same thing at bottom of valley

Textbook §4.1

Suppose f has a local minimum at c.

(the case for maximum is in the book pg 279)

$$f(c) \le f(c+h) \text{ for all } h \text{ close to zero}$$

Thus
$$f(c+h) - f(c) \geq 0$$

Case $h > 0$ then $\dfrac{f(c+h) - f(c)}{h} \geq 0$ since

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = \lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h} \geq 0$$

Case $h < 0$ then $\dfrac{f(c+h) - f(c)}{h} \le 0$

(note the inequality changes direction since $h < 0$)

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = \lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h} \leq 0$$

Therefore $f'(c) \geq 0$ and $f'(c) \leq 0$ so $f'(c) = 0$.

Rolles Theorem

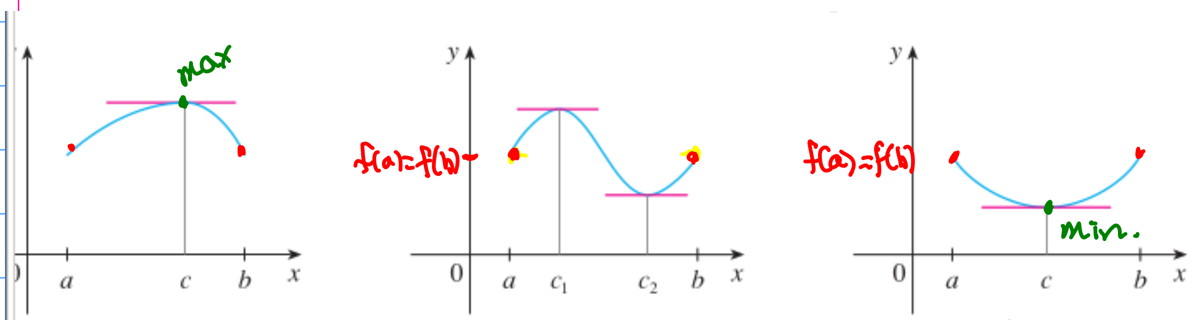
Rolle's Theorem Let f be a function that satisfies the following three hypotheses:

hypothesis

1. f is continuous on the closed interval $[a, b]$. ✓
2. f is differentiable on the open interval (a, b).
3. $f(a) = f(b)$

conclusion

Then there is a number c in (a, b) such that $f'(c) = 0$.



If f goes up then down there is a maximum

both max and min

If f goes down then up there is a minimum.

☑ Case $f(x)$ is constant (goes neither up nor down).

$f'(x) = 0$ Take any $c \in (a,b)$ then $f'(c) = 0$

☑ Case $f(x)$ goes up then down. So there is some point $x \in (a,b)$ where $f(x) > f(a)$. By Extreme value theorem the function has a maximum. The maximum does not occur at either endpoint since $f(x) > f(a)$. Let c be where the maximum occurs. Then $c \in (a,b)$. By Fermat's theorem if f is differentiable at the maximum then $f'(c) = 0$.

☑ Case $f(x)$ goes down then up. So there is some point $x \in (a,b)$ where $f(x) < f(a)$. By Extreme value theorem the function has a minimum. The minimum does not occur at either endpoint since $f(x) < f(a)$. Let c be where the minimum occurs. Then $c \in (a,b)$. By Fermat's theorem if f is differentiable at the minimum then $f'(c) = 0$.

☑ **The Mean Value Theorem** Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

$$\boxed{1} \qquad f'(c) = \frac{f(b) - f(a)}{b - a}$$

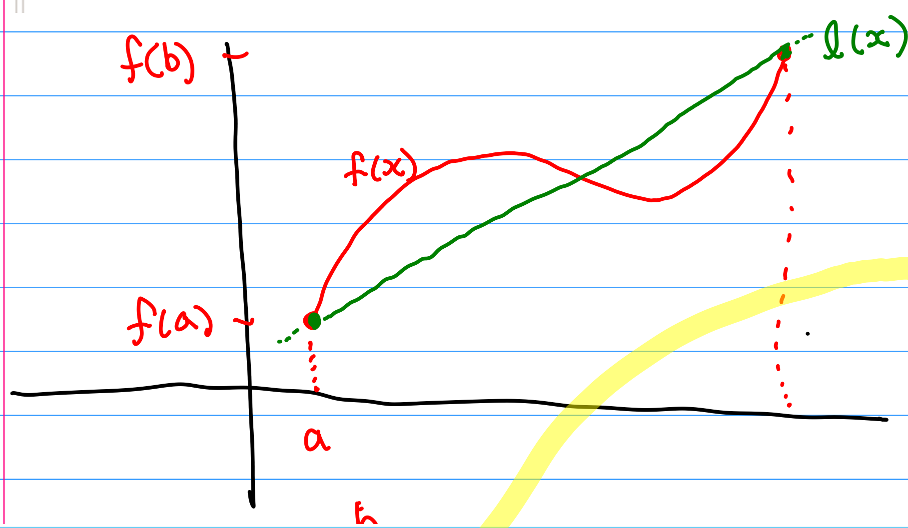
or, equivalently,

$$\boxed{2} \qquad f(b) - f(a) = f'(c)(b - a)$$

Eq of a line. (point-slope)

$$y - f(a) = \frac{f(b)-f(a)}{b-a}(x-a)$$

$$l(x) = f(a) + \frac{f(b)-f(a)}{b-a}(x-a)$$



$F(x) = f(x) - l(x)$

$F(a) = f(a) - l(a) = 0$

$F(b) = f(b) - l(b) = 0$

Note that the line $l(x)$ is cont. and diff.

Rolle's Theorem Let F be a function that satisfies the following three hypot[heses]

1. F is continuous on the closed interval $[a, b]$. ☑ difference of cont. func. is cont.
2. F is differentiable on the open interval (a, b). ☑ difference of diff. func. is diff.
3. $f(a) = f(b)$ ☑

Then there is a number c in (a, b) such that $F'(c) = 0$.

So $F'(c) = 0$.

What is F' that the conclusion...

$$\ell(x) = f(a) + \frac{f(b)-f(a)}{b-a}(x-a) \qquad \ell'(x) = 0 + \frac{f(b)-f(a)}{b-a}(1-0)$$

$$F(x) = f(x) - \ell(x) = f(x) - \left(f(a) + \frac{f(b)-f(a)}{b-a}(x-a)\right)$$

$$F'(x) = f'(x) - \ell'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$$

Therefore

$$F'(c) = f'(c) - \frac{f(b)-f(a)}{b-a} = 0 \quad \text{so} \quad f'(c) = \frac{f(b)-f(a)}{b-a}$$

multiply.

Finally

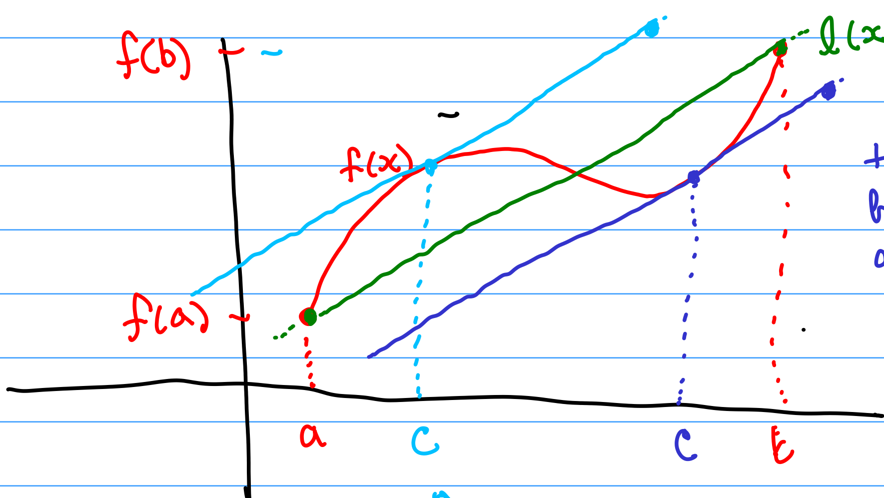
$$f(b) - f(a) = f'(c)(b-a)$$ mean value theorem

1 $\qquad f'(c) = \dfrac{f(b) - f(a)}{b - a}$

or, equivalently,

2 $\qquad f(b) - f(a) = f'(c)(b - a)$

Note



slope of line is $f'(c)$

tangent at $(c, f(c))$ has the same slope as $\ell(x)$

another c where slopes agree.

Theorem If $\boxed{f'(x)=0}$ for $x \in (a,b)$ then $f(x)$ is const.

Let $x_1, x_2 \in (a,b)$ we need to show $f(x_1)=f(x_2)$.

Suppose $x_1 < x_2$ (relabel if not)

Then apply mean value theorem.

The Mean Value Theorem Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval ~~[a, b]~~. $[x_1, x_2]$
2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

1
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

2
$$f(\underset{x_2}{b}) - f(\underset{x_1}{a}) = f'(c)(\underset{x_2}{b} - \underset{x_1}{a})$$

since $x_1, x_2 \in (a,b)$ and f is differentiable on the open interval so f is cont. of (a,b) and thus on $[x_1, x_2]$.

Thus

$$f(x_2) - f(x_1) = \underbrace{f'(c)}_{\text{but } f'(c)=0 \text{ by assumption}}(x_2 - x_1) = 0$$

It follows that $f(x_2) = f(x_1)$.