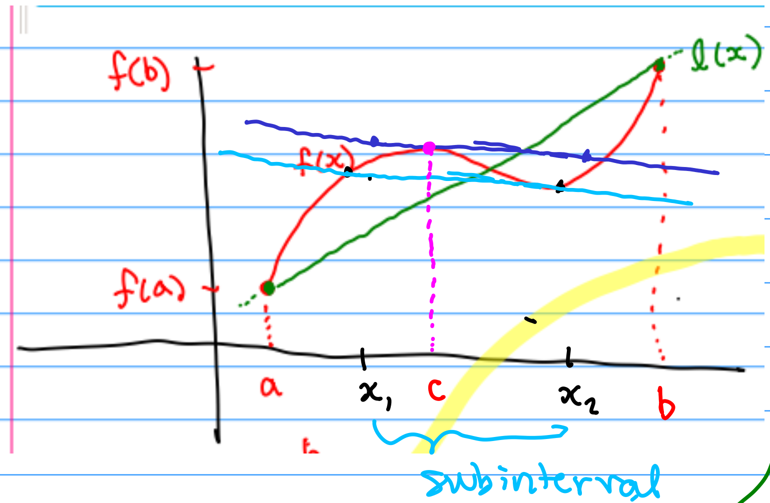

mean value theorem: There is $c \in (x_1, x_2)$ so that

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

or

$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1)$$

If f' is always zero then the right side is zero.

That means $f(x_2) = f(x_1)$ or in other words f is const.

Note: c depends of x_1 and x_2. Since the secant line depends on x_1 and x_2, so the point c where f' has the same slope as the secant line depends on x_1 and x_2.

But $f'(c) = 0$ no what c is so $f'(c)$ does not depend on x_1 and x_2 even though c does.

5 Theorem If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b).

corollary means almost the same as previous Theorem...

7 Corollary If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + c$ where c is a constant.

PROOF Let $F(x) = f(x) - g(x)$. Then

$$F'(x) = f'(x) - g'(x) = 0$$

for all x in (a, b). Thus, by Theorem 5, F is constant; that is, $f - g$ is constant. ■

If the difference between two things is zero those two things were equal, and conversely.

32. If $f'(x) = c$ $\leftarrow g'(x)$ (c a constant) for all x, use Corollary 7 to show that $f(x) = \underbrace{cx + d}$ for some constant d.

$g(x) = cx + d \leftarrow$ const

$g'(x) = c \leftarrow$ const

Then $f'(x) = g'(x)$ for all x

By the Corollary $f - g$ is constant.

In symbols there is a constant b such that

$$f(x) - g(x) = b \quad \text{for all } x$$

Then

$$f(x) = g(x) + b = cx + d + b = cx + \underbrace{(d+b)}_{\text{different constant ?}}$$

Polish the homework answer.

32. If $f'(x) = c$ $\leftarrow g'(x)$ (c a constant) for all x, use Corollary 7 to show that $f(x) = cx + d$ for some constant d.

Proof: Let $g(x) = cx$

then $g'(x) = c \leftarrow$ const

Therefore $f'(x) = g'(x)$ for all x

By the Corollary $f - g$ is constant.

In symbols there is a constant d such that

$$f(x) - g(x) = d \quad \text{for all } x$$

Then

$$f(x) = g(x) + d = cx + d = cx + d$$

for some constant d.

Use Corollary 7 to ...

35. Prove the identity $\arcsin \frac{x-1}{x+1} = 2\arctan\sqrt{x} - \frac{\pi}{2}$.

Take derivate of both sides and show the derivatives are equal. Then check that the constant is really $-\pi/2$.

$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}\left(\frac{x-1}{x+1}\right) = \frac{1\cdot(x+1) - 1\cdot(x-1)}{(x+1)^2}$

$$\frac{d}{dx} \arcsin \frac{x-1}{x+1} = \frac{1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}} \cdot \frac{d}{dx}\left(\frac{x-1}{x+1}\right)$$

$$= \frac{1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}} \cdot \frac{2}{(x+1)^2} = \frac{1}{\sqrt{(x+1)^2-(x-1)^2}} \cdot \frac{2}{x+1}$$

$$= \frac{1}{\sqrt{x^2+2x+1-(x^2-2x+1)}} \cdot \frac{2}{x+1} = \frac{1}{\sqrt{4x}} \cdot \frac{2}{x+1} = \boxed{\frac{1}{\sqrt{x}(x+1)}}$$

$$\frac{d}{dx}\left(2\arctan\sqrt{x} - \frac{\pi}{2}\right) = 2\frac{1}{1+(\sqrt{x})^2} \frac{d}{dx}\sqrt{x} - 0$$

$$= 2 \frac{1}{1+x} \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{\sqrt{x}(1+x)}}$$

same.

So I know $\arcsin \frac{x-1}{x+1}$ and $2\arctan\sqrt{x}$ differ by a const.

evaluate these functions at any point to find const.

Try $x=1$. Then $\arcsin \frac{1-1}{1+1} = \arcsin 0 = 0$

$$2\arctan\sqrt{1} = 2\cdot\frac{\pi}{4} = \frac{\pi}{2}$$