Mean Value Theorem:

The Mean Value Theorem Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

$$\boxed{1} \qquad f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

$$\boxed{2} \qquad f(b) - f(a) = f'(c)(b - a)$$

(says something about the original funct.)

Eq of

$y - f(a) =$

$l(x) = f($

how the derivative is related to the original function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

limit is complication, but anyway gives the derivative in term of the original function

Def of Increasing: A function f is (strictly) increasing on $[a,b]$ if $x_1, x_2 \in [a,b]$ with $x_1 < x_2$ implies $f(x_1) < f(x_2)$

Def of non-decreasing A function f is non-decreasing on $[a,b]$ if $x_1, x_2 \in [a,b]$ with $x_1 \leq x_2$ implies $f(x_1) \leq f(x_2)$.

Mean value theorem

$$\underbrace{f(x_2) - f(x_1) = f'(c)(x_2 - x_1)}$$

Def of increasing: A function f is (strictly) increasing on $[a,b]$ if $x_1, x_2 \in [a,b]$ with $x_1 < x_2$ implies $f(x_1) < f(x_2)$

$f(x_2) > f(x_1)$ if and only if $f(x_2) - f(x_1) > 0$

$\Longleftrightarrow$

$x_2 > x_1$ if and only if $x_2 - x_1 > 0$

Suppose $f'(c) > 0$ then $f'(c)(x_2 - x_1) > 0$

and so $f(x_2) > f(x_1)$ which means the function f is increasing...

Why are increasing functions important? Reason is because the preserve inequalities:

$$f(x) = \sqrt{x} \qquad f'(x) = \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

Is this derivative positive? Yes because $\sqrt{x}$ means the positive root.

We infer $\sqrt{\cdot}$ is an increasing function... Thus $\sqrt{\cdot}$ preserves inequalities: Example

$1 < 2$ take $\sqrt{\cdot}$ on both sides

$\sqrt{1} < \sqrt{2}$ still less than...

Note that a decreasing function reverses an inequality

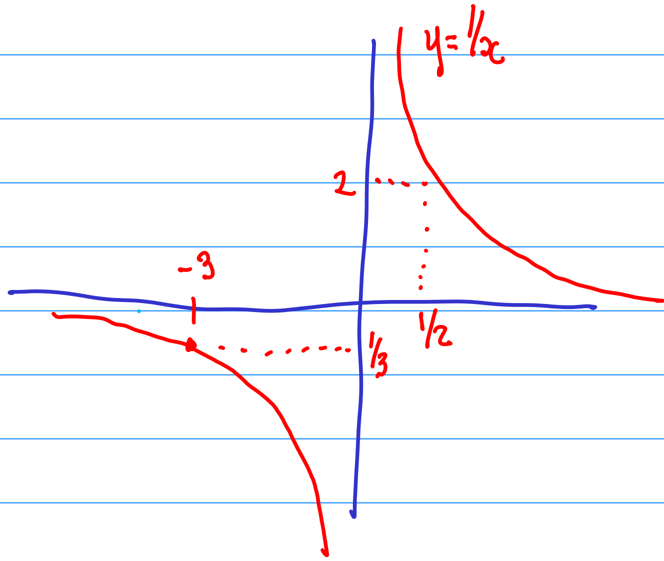
$g(x) = \frac{1}{x}$ $g'(x) = \frac{d}{dx} x^{-1} = -1x^{-2} = \frac{-1}{x^2}$

derivative is negative...

Then $g(x)$ is decreasing... on any interval $[x_1, x_2]$ in its domain..

$1 < 2$ apply g to both sides

$g(1) > g(2)$ $\frac{1}{1} > \frac{1}{2}$



$-3 < 2$

but $\frac{1}{-3} < \frac{1}{2}$

didn't change direction

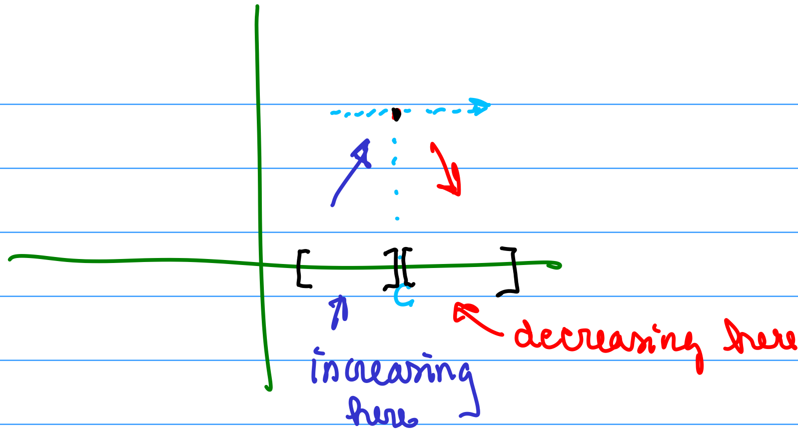
What went wrong?
The interval $[-3, 2]$ is not in the domain of $\frac{1}{x}$. and the derivative $g'(x)$ wasn't negative on that whole interval, or even defined.

The First Derivative Test Suppose that c is a critical number of a continuous function f.

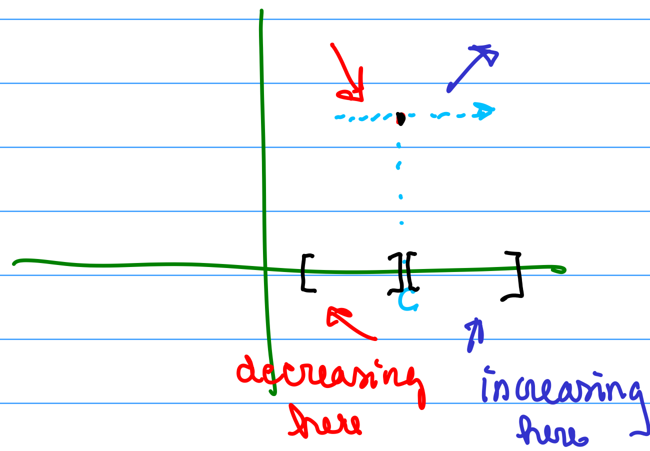
(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

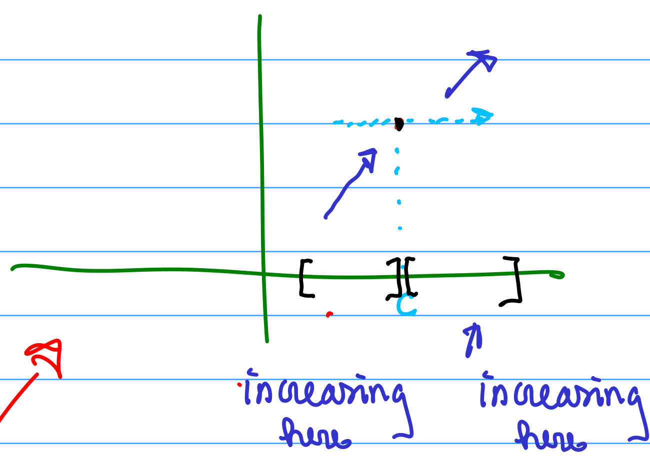
(c) If f' is positive to the left and right of c, or negative to the left and right of c, then f has no local maximum or minimum at c.



then the value $f(c)$ is a maximum.



then the value $f(c)$ is a minimum



then the value $f(c)$ is neither a minimum or max.

example of this

$$f(x) = (x-1)^3 = x^3 - 3x^2 + 3x - 1$$

$$f'(x) = 3(x-1)^2 \frac{d}{dx}(x-1) = 3(x-1)^2$$ positive derivative...

$f'(x) = 0$ means $3(x-1)^2 = 0$ so $x = 1$ is a critical value of f.

But neither a minimum of maximum.

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julia> using Plots

julia> f(x)=(x-1)^3
f (generic function with

julia> plot(f,0:0.1:2)
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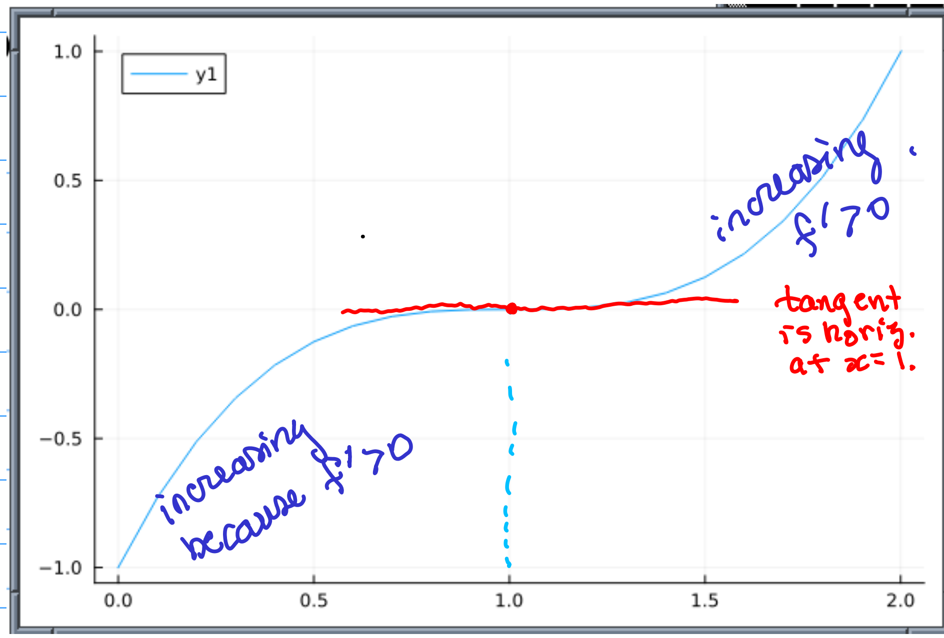

Idea ... don't plot accurate graphs with a computer but draw a sketch (cartoon) of the graph which exagerates and draws attention to essential features.

- min and max
- where increasing and decreasing.
- horiz. and vert. asymptotes

one more thing

- concavity.

Definition If the graph of f lies above all of its tangents on an interval I, then it is called **concave upward** on I. If the graph of f lies below all of its tangents on I, it is called **concave downward** on I.

Figure 7 shows the graph of a function that is concave upward (abbreviated CU) on the intervals (b, c), (d, e), and (e, p) and concave downward (CD) on the intervals (a, b), (c, d), and (p, q).