Look for review material for the final exam on the website. I will post something over the weekend.

Next week

Monday — review

Tuesday — review

Friday — Dec 12 Final here at 10:15 am.

WRB2003

Section 5.5 more examples.

Look for compositions of functions and functions whose derivatives also appear to help guess a change of variables that makes it possible to guess the antiderivative.

69. $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

idea use the chain rule in reverse...

Change of variables for integrals (Chain rule in reverse)

4 The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int_a^b f(\overbrace{g(x))\,g'(x)}^{\text{chain...}}\, dx = \int_{g(a)}^{g(b)} f(u)\, du$$

69. $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

$\ln x$ function composed with $\sqrt{\ }$

$\frac{1}{x}$ is the derivative of $\ln x$

Try change of variables $u = \ln x$ to see if it simplifies the integral so I can guess the antiderivative...

$g(x) = \ln x$

$u = \boxed{\ln x}$ then $\frac{du}{dx} = \frac{1}{x}$ or $du = \boxed{\frac{1}{x}\,dx}$

$$\ldots = \int_e^{e^4} \frac{1}{\sqrt{\ln x}} \cdot \boxed{\frac{1}{x}\,dx} = \int_{\ln e}^{\ln e^4} \frac{1}{\sqrt{u}}\, du = \int_1^4 u^{-1/2}\, du$$

$$= 2u^{1/2}\Big|_1^4 = 2\cdot 4^{1/2} - 2\cdot 1^{1/2} = 4-2=2.$$

30. $\displaystyle\int \frac{\sec^2 x}{\tan^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx$

is this easier to guess antiderivative,

$\sec x = \dfrac{1}{\cos x}$

$\tan x = \dfrac{\sin x}{\cos x}$

Could these be useful.

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

functions whose derivatives also appear

$\dfrac{d\tan x}{dx} = \sec^2 x$

Try a change of variables

$u = \tan x \qquad \dfrac{du}{dx} = \sec^2 x \qquad du = \sec^2 x\,dx$

$$\ldots = \int \frac{1}{(\tan x)^2}\sec^2 x\,dx = \int \frac{1}{u^2}\,du = \int u^{-2}\,du$$

$$= \frac{u^{-2+1}}{-2+1} + C = -\frac{1}{\tan x} + C = -\cot x + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$\int \csc^2 x\,dx = -\cot x + C$ since $\csc x = \dfrac{1}{\sin x}$ the formula agrees.

Look for compositions of functions and functions whose derivatives also appear to help guess a change of variables that makes it possible to guess the antiderivative.

71. $\displaystyle\int_0^1 \frac{e^z+1}{e^z+z}\,dz$

$\frac{d}{dz}(e^z+z) = e^z+1$

$g(z) = e^z + z$

$u = e^z + z$ $\qquad \frac{du}{dz} = e^z+1 \qquad du = (e^z+1)\,dz$

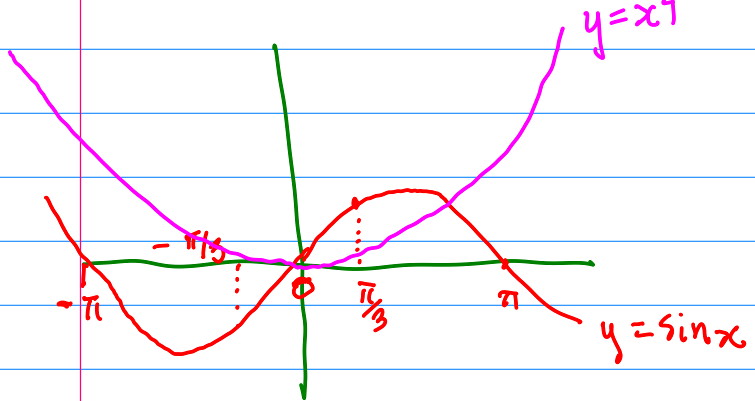
$$\int_0^1 \frac{1}{e^z+z}(e^z+1)\,dz = \int_{e^0+0}^{e^1+1} \frac{1}{u}\,du = \ln u \Big|_1^{e+1}$$

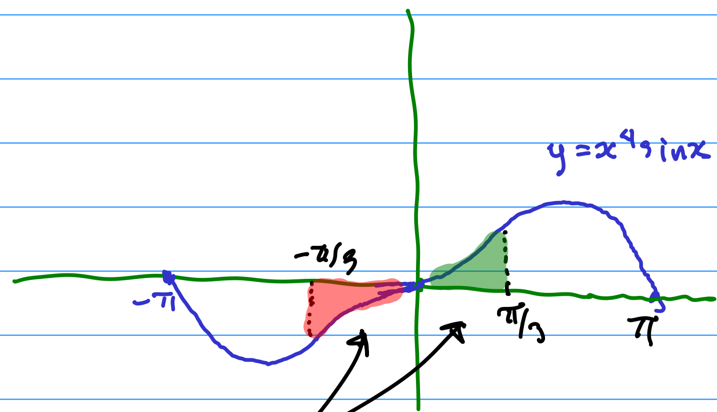
$$= \ln(e+1) - \underbrace{\ln 1}_{0} = \ln(e+1)$$

the function is odd ...

66. $\displaystyle\int_{-\pi/3}^{\pi/3} x^4 \sin x\,dx = 0$

interval of integration is symmetric..





By the symmetry these two areas are the same... but since say is counted negative they cancel out

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x\,dx = \int_{-\pi/3}^{0} x^4 \sin x\,dx + \int_{0}^{\pi/3} x^4 \sin x\,dx$$

make a change of variables

$$\int_{-\pi/3}^{0} x^4 \sin x\,dx$$

$g(x) = -x$

$u = -x \qquad \frac{du}{dx} = -1 \qquad -du = dx$

$x = -u$

$$= \int_{\pi/3}^{-0} (-u)^4 \sin(-u)\,(-du)$$

$$= -\int_{\pi/3}^{0} u^4 (-\sin u)\,du = \int_{\pi/3}^{0} u^4 \sin u\,du$$

& switch endpoints of the directed interval another minus sign.

$$= -\int_{0}^{\pi/3} u^4 \sin u\,du$$

Therefore

$$\int_{-\pi/3}^{0} x^4 \sin x\,dx = -\int_{0}^{\pi/3} u^4 \sin u\,du$$

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x\,dx = \boxed{\int_{-\pi/3}^{0} x^4 \sin x\,dx} + \boxed{\int_{0}^{\pi/3} x^4 \sin x\,dx}$$

$$= -\int_{0}^{\pi/3} u^4 \sin u\,du + \boxed{\int_{0}^{\pi/3} x^4 \sin x\,dx} = 0.$$