Math 181 Final Review Version E

1. Precisely define $\lim_{x\to a} f(x) = L$ using inequalities in terms of δ and ϵ.

2. Define the derivative $f'(x)$ of a function $f(x)$ using limits.

3. Suppose $2x\sin y + y\sin x = 3$. Find dy/dx by implicit differentiation.

4. Define the integral $\int_a^b f(x)dx$ of a function $f(x)$ using limits.

5. Find the following limits:

(i) $\lim_{x\to 3} \frac{x^2-x-6}{x-3}$

(ii) $\lim_{x\to\infty} \frac{x^2+3x-1}{3x^2-1}$

(iii) $\lim_{t\to 0} \frac{1-e^{-3t}}{t}$

6. Find the following derivatives:

(i) $\frac{d}{dx}\arctan(-2x)$

(ii) $\frac{d}{dx}\left(\frac{x}{x^2+5}\right)$

(iii) $\frac{d}{dx}|\sin(5-2x)|$

7. Find the following antiderivatives:

(i) $\int (3x^3+x^2)\,dx$

(ii) $\int x^6\cos(x^7+1)\,dx$

(iii) $\int x\sqrt{x-2}\,dx$

8. Compute the following areas:

(i) $\int_1^2 x^{-3}dx$

(ii) $\int_0^{\pi/2} \sin 2x\,dx$

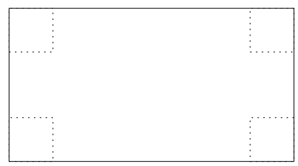
(iii) $\int_1^4 \frac{1}{x+2}\,dx$

9. Solve the following story problems:

(i) The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 5 cm/s. When the length is 12 cm and the width is 4 cm, how fast is the area of the rectangle increasing?

(ii) A street light is mounted at the top of a 12-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 3 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

(iii) A box with an open top is to be constructed from a single sheet of metal 3 inches long and 2 inches wide by cutting out the corners and folding up the sides.



Find the height of the box with the maximimal volume.

10. Answer the following true/false questions:

(i) $\lim_{\theta \to 0} \frac{\cos \theta}{\theta} = 1.$

(ii) If f is continuous at a, then f is differentiable at a.

(iii) If f is continuous on $[a, b]$, then the integral $\int_a^b f(x)dx$ exists.

3. Suppose $2x\sin y + y\sin x = 3$. Find dy/dx by implicit differentiation.

$$\frac{d}{dx}\left(2x\sin y + y\sin x\right) = \frac{d}{dx}3$$

product product

$$\left(\frac{d}{dx}2x\right)\sin y + 2x\left(\frac{d}{dx}\sin y\right) + \frac{dy}{dx}\sin x + y\left(\frac{d}{dx}\sin x\right) = 0$$

$$2\sin y + 2x\cos y\, y' + y'\sin x + y\cos x = 0$$

solve for y'

$$y'(2x\cos y + \sin x) = -2\sin y - y\cos x$$

$$y' = \frac{-2\sin y - y\cos x}{2x\cos y + \sin x}$$