

Math 181 Sample Midterm Version D

1. For the function $y = g(x)$ depicted in the graph state the value of each quantity, if it exists. If it does not, explain why.

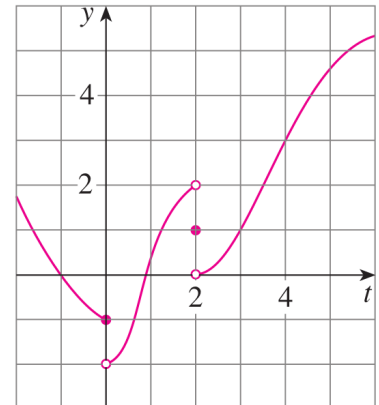
(i) $\lim_{t \rightarrow 0^-} g(t)$

(ii) $\lim_{t \rightarrow 0^+} g(t)$

(iii) $\lim_{t \rightarrow 0} g(t)$

(iv) $g(2)$

(v) $\lim_{t \rightarrow 4} g(t)$



2. Precisely define $\lim_{x \rightarrow a} f(x) = L$ using inequalities in terms of δ and ϵ .

3. Show $\lim_{x \rightarrow 5} \frac{1}{x} = \frac{1}{5}$ using the δ - ϵ definition of limit.

4. Given that

$$\lim_{x \rightarrow 2} f(x) = 9, \quad \lim_{x \rightarrow 2} g(x) = -2 \quad \text{and} \quad \lim_{x \rightarrow 2} h(x) = 0$$

find each limit, if it exists. If it does not exist, explain why.

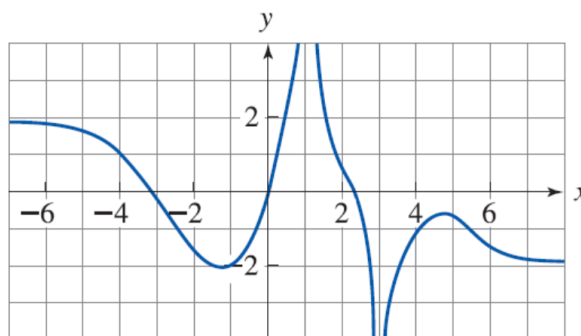
(i) $\lim_{x \rightarrow 2} (f(x) + 4g(x))$

(ii) $\lim_{x \rightarrow 2} (g(x))^3$

(iii) $\lim_{x \rightarrow 2} \sqrt{f(x)}$

(iv) $\lim_{x \rightarrow 2} \frac{2f(x)}{g(x)}$

5. For the function $y = f(x)$ whose graph is



state the value of each limit, if it exists. If the limit is infinity, write ∞ or $-\infty$ as appropriate. If the limit does not otherwise exist, write DNE.

(i) $\lim_{x \rightarrow \infty} f(x)$

(ii) $\lim_{x \rightarrow \infty} f(x)$

(iii) $\lim_{x \rightarrow -\infty} f(x)$

(iv) $\lim_{x \rightarrow 1} f(x)$

(v) $\lim_{x \rightarrow 3} f(x)$

6. Find the following limits:

(i) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$

(ii) $\lim_{x \rightarrow \infty} \frac{1 + x - 4x^2}{2 + x^2}$

(iii) $\lim_{h \rightarrow 0} \frac{\sqrt{3+h} - \sqrt{3}}{h}$

(iv) $\lim_{t \rightarrow 0} \frac{(\sin 5t)^2}{t^2}$

7. Define the derivative $f'(x)$ of a function $f(x)$ using limits.
8. Use the limit definition to explain why the derivative of $f(x) = x^2$ is $f'(x) = 2x$.
9. Suppose g is differentiable and $f(x) = g(x)/x$. Use the limit definition of derivative to explain why $f'(x) = g'(x)/x - g(x)/x^2$ when $x \neq 0$. No points will be awarded for applying the quotient rule.

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10. State the following derivative rules from memory:

$$\frac{d}{dx} \sin x =$$

$$\frac{d}{dx}(fg)(x) =$$

$$\frac{d}{dx} \cos x =$$

$$\frac{d}{dx}(f \circ g)(x) =$$

$$\frac{d}{dx} \tan x =$$

$$\frac{d}{dx} \left(\frac{f}{g} \right) (x) =$$

$$\frac{d}{dx}x^\alpha =$$

$$\frac{d}{dx} \frac{1}{x^\beta} =$$

$$\frac{d}{dx} \ln x =$$

$$\frac{d}{dx} \arccos x =$$

$$\frac{d}{dx} \sec x =$$

$$\frac{d}{dx} \log_b x =$$

$$\frac{d}{dx} \arcsin x =$$

$$\frac{d}{dx} \csc x =$$

$$\frac{d}{dx} e^x =$$

$$\frac{d}{dx} \arctan x =$$

$$\frac{d}{dx} \cot x =$$

$$\frac{d}{dx}a^x =$$

11. Use the rules of calculus to compute the following derivatives:

(i) $\frac{d}{dx}((1 + \sqrt{x}) \sin x)$

(ii) $\frac{d}{dx} \arccos(1 + x^2)$

(iii) $\frac{d}{dx} \left(\frac{x^3 - 2}{x^2 + 1} \right)$

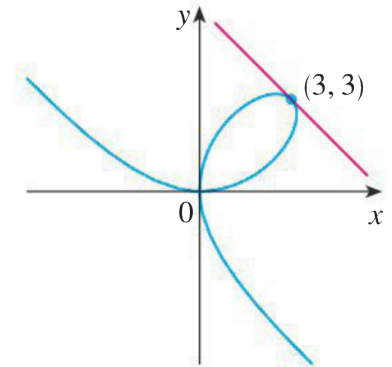
(iv) $\frac{d}{dx} \log_{10}(3 + \sin x)$

(v) $\frac{d}{dx} 5^x$

12. Consider the curve defined by the equation $x^3 + y^3 = 6xy$.

(i) Use implicit differentiation to find y' in terms of x and y .

(ii) Find the equation of the line tangent to this curve at the point $(3, 3)$.



(iii) At what point in the first quadrant is the tangent line horizontal?

- 13.** A man walks along a straight path at a speed of 5 ft/s. A searchlight is located on the ground 3 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 4 ft from the point on the path closest to the searchlight?

