Math 285 Lecture 1

1.1 Introduction.

What is a differential equation?

Zill, A first course in differential equations any edition...

An equation with terms that are derivatives.

Derivatives tell rates of change...

Example: $$y'' + 5(y')^2 - 4y = e^x$$

here $y = y(x)$ is a function of x and $y'(x) = \frac{dy(x)}{dx}$ is the first derivative of y, ... so forth...

Classification with respect to order.

what is the highest derivative in the equation?

$$\underbrace{y''}_{\text{second derivative}} + 5(\underbrace{y'}_{\text{first derivative}})^2 - 4y = e^x$$

conclusion: this is a second order derivative.

Curious fact: all differential equation of any order can be written as a system of first order differential equation:

$$y'' + 5(y')^2 - 4y = e^x$$

Let $v = y'$. then $v' = y''$.

$$\begin{cases} v' + 5(v)^2 - 4y = e^x \\ y' = v \end{cases}$$

Classification with respect to linearity.

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdot\ \cdot\ \cdot + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x).$$

General n-th order linear equation

$$y'' + 5(y')^2 - 4y = e^x$$

$g(x) = e^x$

?

$a_2(x) = 1$.

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = g(x)$$

$a_0(x) = -4$

Could try to take square root. But this doesn't help...

Conclusion. The equation was not linear...

Linearity is about how the equation initially appears, even though there might be some crazy way to transform it to make it linear.

$y' + 5y = 7$ ⇐ linear..