$$\frac{dy}{dx} = \underbrace{g(x)h(y)}_{f(x,y)\text{: has a special form}}$$

① $\frac{1}{h(y)}\frac{dy}{dx} = g(x)$ — divide ✓

② $\int \frac{1}{h(y)}\frac{dy}{\cancel{dx}}\cancel{dx} = \int g(x)\,dx$ — integrate both sides $\int \cdots dx$ ✓

③ $\int \frac{1}{h(y)}dy = \int g(x)\,dx$ — use change of variables...

Example :
$$\frac{dy}{dx} = -\frac{x}{y}$$

$g(x) = -x$, $h(y) = \frac{1}{y}$

$$y\frac{dy}{dx} = -x$$

$$\int y \frac{dy}{\cancel{dx}}\cancel{dx} = \int -x\,dx$$

$$\int y\,dy = -\int x\,dx$$

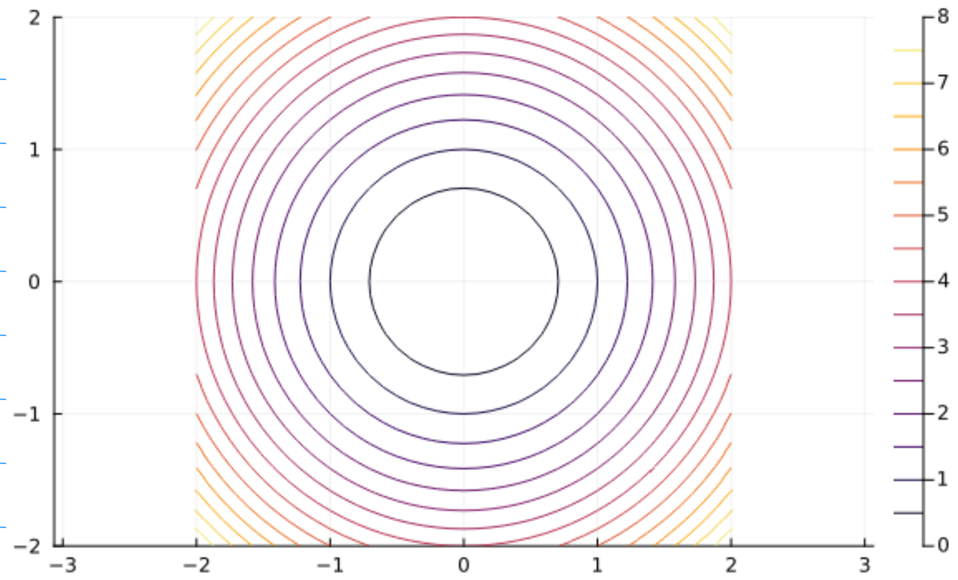
$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + c_1$$

$$y^2 = -x^2 + \underbrace{2c_1}_{c_2}$$

Thus $x^2 + y^2 = c_2$ general solutions, can solve for c_2 using an initial condition to obtain a unique solution...

```
julia> using Plots
```

```
contour(-2:0.1:2,-2:0.1:2,(x,y)->x^2+y^2,aspect_ratio=1)
```



Example

$$\frac{dy}{dx} = x\sqrt{1-y^2}$$

$g(x)=x, \quad h(x)=\sqrt{1-y^2}$

shortcut

$$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = x$$

$$\int \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}\,dx = \int x\,dx$$

$$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int x\,dx = \frac{1}{2}x^2 + C_1$$

Trig subs

$$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int \frac{1}{\sqrt{1-\sin^2\theta}}\cos\theta\,d\theta = \int \frac{\cos\theta}{\sqrt{\cos^2\theta}}\,d\theta$$

$y = \sin\theta \qquad dy = \cos\theta\,d\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\theta = \arcsin y$

this is +1 when $\cos\theta > 0$
-1 when $\cos\theta < 0$

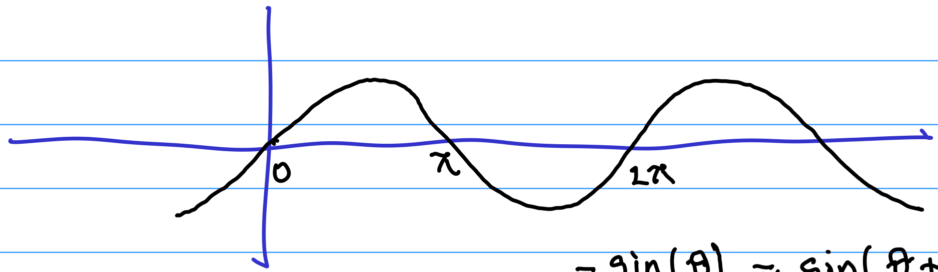
$$\int \frac{\cos\theta}{\sqrt{\cos^2\theta}}\,d\theta = \int \frac{\cos\theta}{|\cos\theta|}\,d\theta = \int \pm 1\,d\theta = \pm\theta$$

$$\pm\theta = \frac{1}{2}x^2 + c_1$$

$$\pm \arcsin y = \frac{1}{2}x^2 + c_1$$

$$y = \pm\sin\left(\frac{1}{2}x^2 + c_1\right)$$

solve for the constant c_1 and the $\pm$ based on the initial condition



$$-\sin(\theta) = \sin(\theta + \pi)$$

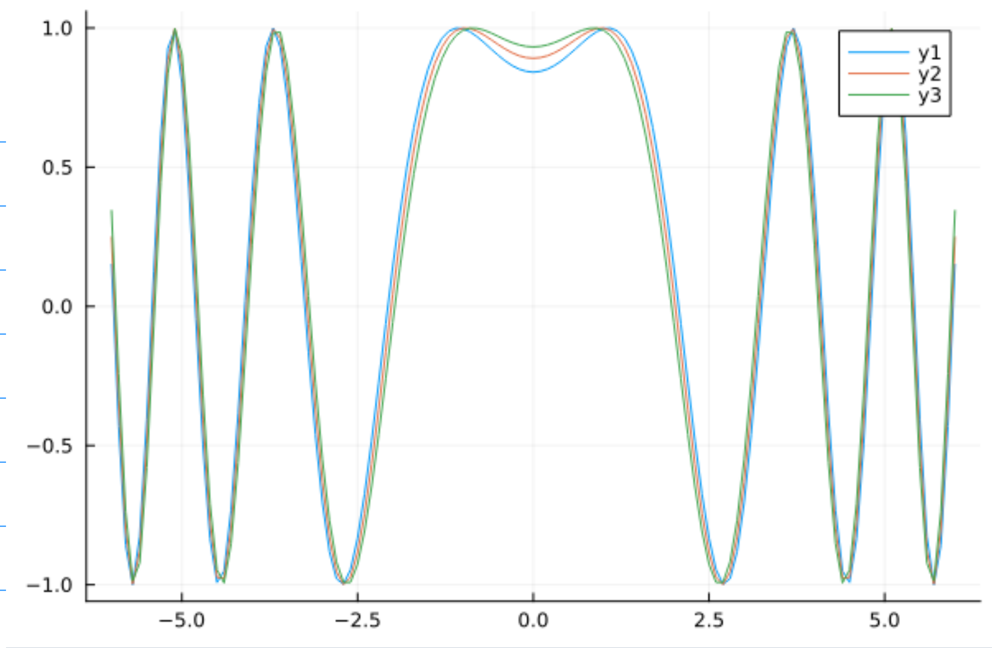
since there is a dependency between c_1 and the $\pm$ we can swallow the $\pm$ into the c_1 by shifting c_1 by π.

$$y = \sin\left(\frac{1}{2}x^2 + c_2\right)$$ general solution...

```
plot(-6:0.1:6,x->sin(1/2*x^2+1))

plot!(-6:0.1:6,x->sin(1/2*x^2+1.1))

plot!(-6:0.1:6,x->sin(1/2*x^2+1.2))
```




```
plot(-6:0.1:6,[x->sin(1/2*x^2+c) for c=1:0.5:3])
```

Plot a bunch of different c's