$$\frac{dx}{dy} - \frac{4}{y}x = 4y^5$$

- original idea was solve $\frac{dx}{dy} - \frac{4}{y}x = 0$ which is seperable and use the resulting solution to simplify the original problem

$$\underbrace{\mu \frac{dx}{dy} - \mu \frac{4}{y}x}_{} = \mu 4y^5$$

so this becomes the derivative of a product...

$$\mu = e^{\int -\frac{4}{y}dy} = e^{-4\ln|y|} = \left(e^{\ln|y|}\right)^{-4} = |y|^{-4} = \frac{1}{|y|^4} = \frac{1}{y^4}$$

since 4 is even the $\pm$ in y goes away just by taking a power

$$\underbrace{\frac{1}{y^4}\frac{dx}{dy} - \frac{4}{y^5}x}_{} = 4y$$

$$\frac{d}{dy}\left(\frac{1}{y^4}x\right) = 4y$$

now integrate and use fund theorem of calculus

$$\int \frac{d}{dy}\left(\frac{1}{y^4}x\right)dy = \int 4y\,dy$$

$$\frac{1}{y^4}x = 2y^2 + C.$$

This defines a solution $y(x)$ or $x(y)$ implicitly.

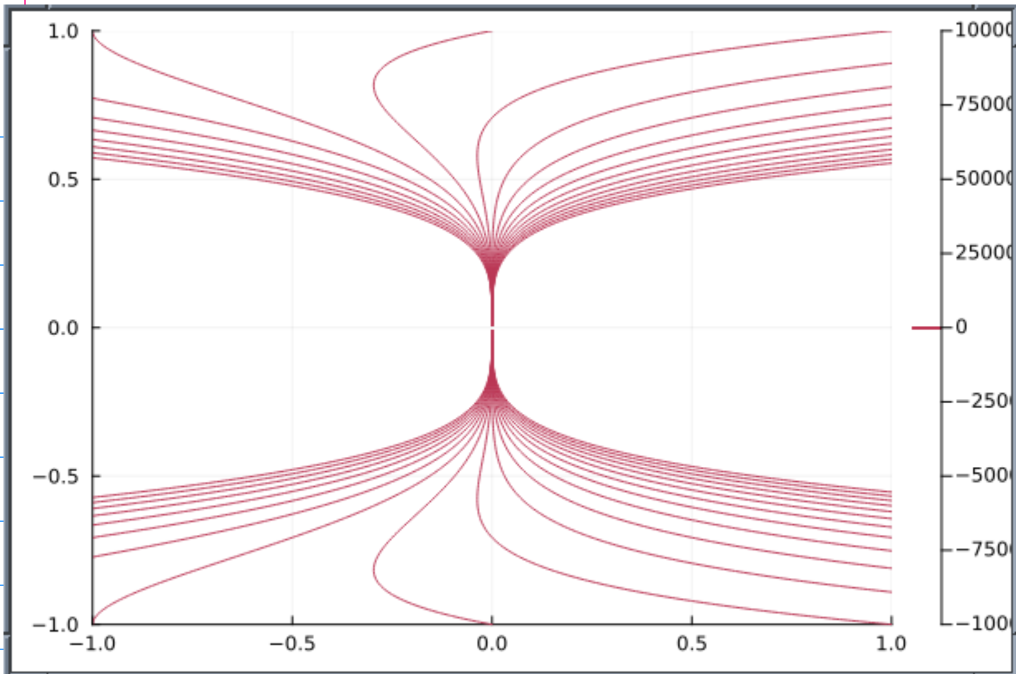
Numerical technique to graph $f(x,y) = C$

$$f(x,y) = \frac{x}{y^4} - 2y^2$$

```
julia> f(x,y)=x/y^4-2*y^2
f (generic function with 1 method)

julia> using Plots
```

```
julia> contour(-1:0.01:1,-1:0.01:1,f,levels=-10:10)
```



Solutions were of the form $f(x,y) = c$ in the above case

What differential equation has this for its solution?

$$\frac{d}{dx} f(x,y) = \frac{d}{dx} c$$

$$M(x,y) = \frac{\partial f}{\partial x}$$

$$\frac{d}{dx} f(x,y) = \underbrace{\frac{\partial f}{\partial x}}_{M} + \underbrace{\frac{\partial f}{\partial y}}_{N} \frac{dy}{dx} = 0$$

$$N(x,y) = \frac{\partial f}{\partial y}$$

$$M + N\frac{dy}{dx} = 0$$

write as a differential (for style)

$$M\,dx + N\,dy = 0$$

$$\underbrace{y}_{M} \, dx - 4(x + y^6)\, dy = 0$$

$M = y \qquad N = -4(x+y^6)$

Question is there a function $f(x,y)$ such that $M = \frac{\partial f}{\partial x}$ and $N = \frac{\partial f}{\partial y}$.

Easy way to check. If $M = \frac{\partial f}{\partial x}$ and $N = \frac{\partial f}{\partial y}$.

Then $\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial^2 f}{\partial x \partial y}$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x \partial y}$$

If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then there is an f so that

$$M = \frac{\partial f}{\partial x} \quad \text{and} \quad N = \frac{\partial f}{\partial y}$$

and the solution to

$$M\,dx + N\,dy = 0$$

is $f(x,y) = c$ where c is a general parameter.