Given $f: X \to Y$.

True or False: $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$ for $S, T \subseteq Y$.

✓ "$\subseteq$" Let $x \in f^{-1}(S \cap T)$ then $f(x) \in S \cap T$
Then $f(x) \in S$ and $f(x) \in T$.
That means $x \in f^{-1}(S)$ and $x \in f^{-1}(T)$.
therefore $x \in f^{-1}(S) \cap f^{-1}(T)$

"$\supseteq$" Let $x \in f^{-1}(S) \cap f^{-1}(T)$
Since all the steps for the "$\subseteq$" inclusion were reversible, then $x \in f^{-1}(S \cap T)$.

**Proposition 1.12** Let $f$ be a function from $X$ into $Y$ and let $g$ be a function from $Y$ into $Z$.

1. If both $f$ and $g$ are one-to-one, then so is $g \circ f$.
2. If both $f$ and $g$ are onto functions, then so is $g \circ f$.
3. If both $f$ and $g$ are bijections, then so is $g \circ f$.

If $f$ is a decreasing function and $g$ is a decreasing function, then is $g \circ f$ also decreasing?

NO!

$\mathbb{N}$ *is Well-Ordered.* Every nonempty subset of $\mathbb{N}$ has a least element. That is, if $A \subset \mathbb{N}$ and $A \neq \emptyset$, then there is an $a_0$ in $A$ such that $a_0 \leq a$ for all $a$ in $A$.

Interesting, because we are interested in making only a minimum number of assumptions as axioms from which to develop the rest of Calculus.

$\mathbb{N}$ well ordered is equivalent to saying that induction can be used to prove $P(n)$ for all $n \in \mathbb{N}$.

*By the end of this chapter, the reader should understand the difference between the real numbers and the rational numbers. In Section 2.1, we note that both are fields with an order relation. However, in Sections 2.2 and 2.3, we show that the real numbers are "complete" whereas the rational numbers are not complete. In Section 2.4, we show that the rational numbers are "countable" but that the real numbers are "uncountable."*