Math 311 Quiz 5 Version A

1. Please fill in the missing blanks to make the theorem correct.

The Integral Test: Suppose f is a __________, __________ function on the half-line $[1, \infty)$. Then the series $\sum_{n=1}^{\infty} f(n)$ converges if and only if the improper integral $\int_1^{\infty} f(x)dx$ converges.

2. Determine if the following series converge or diverge and use the theorems on the other side of this quiz (along with the integral test above if needed) to explain why.

(i) $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

(ii) $\displaystyle\sum_{n=0}^{\infty} \frac{(2n+1)^{3n}}{(3n+1)^{2n}}$

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Please use the following theorems along with the integral test at the top of the other side when explaining your conclusions for question 2 on this quiz.

Theorem 6.9. The series $\sum_1^\infty n^{-p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Theorem 6.11. Suppose $0 \leq a_n \leq b_n$ for $n \geq 0$. If $\sum_0^\infty b_n$ converges, then so does $\sum_0^\infty a_n$. If $\sum_0^\infty a_n$ diverges, then so does $\sum_0^\infty b_n$.

Theorem 6.12. Suppose $\{a_n\}$ and $\{b_n\}$ are sequences of positive numbers and that a_n/b_n approaches a positive, finite limit as $n \to \infty$. Then the series $\sum_0^\infty a_n$ and $\sum_0^\infty b_n$ are either both convergent or both divergent.

Theorem 6.13. Suppose $\{a_n\}$ is sequence of positive numbers.

a. If $a_{n+1}/a_n < r$ for all sufficiently large n, where $r < 1$, then the series $\sum_0^\infty a_n$ converges. On the other hand, if $a_{n+1}/a_n \geq 1$ for all sufficiently large n, then the series $\sum_0^\infty a_n$ diverges.

b. Suppose that $l = \lim_{n\to\infty} a_{n+1}/a_n$ exists. Then the series $\sum_0^\infty a_n$ converges if $l < 1$ and diverges if $l > 1$. No conclusion can be drawn if $l = 1$.

Theorem 6.13. Suppose $\{a_n\}$ is sequence of positive numbers.

a. If $a_n^{1/n} < r$ for all sufficiently large n, where $r < 1$, then the series $\sum_0^\infty a_n$ converges. On the other hand, if $a_n^{1/n} \geq 1$ for all sufficiently large n, then the series $\sum_0^\infty a_n$ diverges.

b. Suppose that $l = \lim_{n\to\infty} a_n^{1/n}$ exists. Then the series $\sum_0^\infty a_n$ converges if $l < 1$ and diverges if $l > 1$. No conclusion can be drawn if $l = 1$.