Math 311 Quiz 3 Version A

1. Please fill in the missing blanks to make the theorem correct.

The Implicit Function Theorem for a System of Equations: Let $F(x, y)$ be an $\mathbf{R}^k$-valued function of class C^1 on some neighborhood of a point $(a, b) \in \mathbf{R}^{n+k}$ and let $B_{ij} = (\partial F_i/\partial y_j)(a, b)$. Suppose that $F(a, b) = 0$ and $\boxed{}$. Then there exist positive numbers r_0 and r_1 such that the following conclusions are valid.

a. For each x in the ball $|x - a| < r_0$ there is a unique y such that $|y - b| < r_1$ and $F(x, y) = \boxed{}$. We denote this y by $f(x)$. In particular, $f(a) = b$.

b. The function f thus defined for $|x - a| < r_0$ is of class $\boxed{}$ and its partial derivatives $\partial f/\partial x_j$ can be computed by differentiating the equations $F(x, f(x)) = 0$ with respect to x_j and solving the resulting linear system for $\partial f_1/\partial x_j, \ldots, \partial f_k/\partial x_j$.

2. Suppose $F(x, y)$ is a C^1 function such that $F(0, 0) = 0$. What conditions on F will guarantee that the equation $F(F(x, y), y) = 0$ can be solved for y as a C^1 function of x near $(0, 0)$?