ELEMENTARY ROW OPERATIONS

1. (Replacement) Replace one row r_i by the sum of itself and a multiple α of $\boxed{\text{another}}$ r_j row.[1] $i \neq j$
2. (Interchange) Interchange two rows. r_i with r_j
3. (Scaling) Multiply all entries in a row by a nonzero constant.

① Replacement (Elimination) $r_i \leftarrow r_i + \alpha r_j$, $i \neq j$

Short hand $[r_i + \alpha r_j]$ implies replace r_i

② Swap rows $r_i \leftrightarrow r_j$

③ Scaling $r_i \leftarrow \alpha r_i$

Short hand: $[\alpha r_i]$

Demonstrate row operations

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 2 \\ -1 & 0 & 7 \end{bmatrix} \begin{matrix} \leftarrow r_1 \\ \leftarrow r_2 \\ \leftarrow r_3 \end{matrix}$$

(Actually use operation 1 to construction the Row Echelon form of A)

$r_2 \leftarrow r_2 - 3r_1$

$i=2$, $\alpha = -3$ $j=1$

why the replacement step is actually an elimination step.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -7 \\ -1 & 0 & 7 \end{bmatrix}$$

$r_3 \leftarrow r_3 + r_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -7 \\ 0 & 2 & 10 \end{bmatrix}$$

$r_3 \leftarrow r_3 + r_2$

Gaussian Elimination — do these steps in a certain order so the zero's made in the earlier steps don't dissapear...

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -7 \\ 0 & 0 & 3 \end{bmatrix}$$

At this point one has factored

$A = LU$

Identifies this as the Eschelon form of A

No need to stop ... could make more zeros ....

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -7 \\ 0 & 0 & 3 \end{bmatrix}$$

$\boxed{r_1 \leftarrow r_1 + r_2}$

change this one.

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & -2 & -7 \\ 0 & 0 & 3 \end{bmatrix}$$

$\boxed{r_1 \leftarrow r_1 + \frac{4}{3} r_3}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -7 \\ 0 & 0 & 3 \end{bmatrix}$$

$r_2 \leftarrow r_2 + \frac{7}{3} r_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Now rescaling

$r_2 \leftarrow -\frac{1}{2} r_2$

$r_3 \leftarrow \frac{1}{3} r_3$

for finding inverse of A

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

reduced row eschelon form