USING ROW REDUCTION TO SOLVE A LINEAR SYSTEM

☑ 1. Write the augmented matrix of the system.

☑ 2. Use the row reduction algorithm to obtain an equivalent augmented matrix in echelon form. Decide whether the system is consistent. If there is no solution, stop; otherwise, go to the next step.

☑ 3. Continue row reduction to obtain the reduced echelon form.

☐ 4. Write the system of equations corresponding to the matrix obtained in step 3.

5. Rewrite each nonzero equation from step 4 so that its one basic variable is expressed in terms of any free variables appearing in the equation.

①

$$1x - 2y + 3z = 6$$
$$2x - 5y + 2z = 1$$
$$-x + 1y - 8z = -3$$

Aug. Matrix

$$\begin{bmatrix} 1 & -2 & 3 & | & 6 \\ 2 & -5 & 2 & | & 1 \\ -1 & 1 & -8 & | & -3 \end{bmatrix}$$

$r_2 \leftarrow r_2 - 2r_1$

$r_3 \leftarrow r_3 + r_1$

$$\begin{bmatrix} 1 & -2 & 3 & 6 \\ 0 & -1 & -4 & -11 \\ 0 & -1 & -5 & 3 \end{bmatrix}$$

$r_3 \leftarrow r_3 - r_2$

row echelon form

$$\begin{bmatrix} 1 & -2 & 3 & 6 \\ 0 & -1 & -4 & -11 \\ 0 & 0 & -1 & 14 \end{bmatrix}$$

Translate the form back

$$1x - 2y + 3z = 6$$
$$-1y - 4z = -11$$
$$-1z = 14$$

if this were also zero then what?

$z = -14$

134
42
176
6

Back Substitution

$$y = 11 - 4z = 11 + 56 = 67$$

$$x = 6 + 2y - 3z = 6 + 134 + 42 = 182$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 182 \\ 67 \\ -14 \end{bmatrix}$$

$$+y + 4z = +11$$
$$y + 4z = 11$$
$$y = 11 - 4z$$

Keep working for reduced row eschelon form

$$\begin{bmatrix} 1 & -2 & 3 & 6 \\ 0 & -1 & -4 & -11 \\ 0 & 0 & -1 & 14 \end{bmatrix}$$

$$r_1 \leftarrow r_1 - 2r_2$$

$$\begin{bmatrix} 1 & 0 & 11 & 28 \\ 0 & -1 & -4 & -11 \\ 0 & 0 & -1 & 14 \end{bmatrix}$$

$$r_1 \leftarrow r_1 + 11r_3$$
$$r_2 \leftarrow r_2 - 4r_3$$

$$\begin{array}{r} 14 \\ 14 \\ 154 \\ 28 \\ \hline 182 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 182 \\ 0 & -1 & 0 & -67 \\ 0 & 0 & -1 & 14 \end{bmatrix}$$

$$28 + 11 \cdot 14 = 182$$
$$-11 - 4 \cdot 14 = -67$$

$$r_2 \leftarrow (-1)r_2$$
$$r_3 \leftarrow (-1)r_3$$

rescaling

reduced row eschelon form...

$$\begin{bmatrix} 1 & 0 & 0 & 182 \\ 0 & 1 & 0 & 67 \\ 0 & 0 & 1 & -14 \end{bmatrix}$$

$$1x = 187$$
$$1y = 67$$
$$1z = -14$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 187 \\ 67 \\ -14 \end{bmatrix}$$