This is a closed-book closed-notes no-calculator-allowed in-class exam. Efforts have been made to keep the arithmetic simple. If it turns out to be complicated, that's either because I made a mistake or you did. In either case, do the best you can and check your work where possible. While getting the right answer is nice, this is not an arithmetic test. It's more important to clearly explain what you did and what you know.

- 1. Indicate in writing that you have understood the requirement to work independently by writing "I have worked independently on this quiz" followed by your signature as the answer to this question.
- 2. Write down the augmented matrix  $\begin{bmatrix} A \mid b \end{bmatrix}$  corresponding to the system of linear equations given by  $\begin{cases} 2x_1 2x_2 + x_4 = 5 \\ x_2 5x_3 5x_4 = 2 \\ -2x_1 + 3x_2 5x_4 = -7 \end{cases}$

but do not solve these equations.

From Exam 1

note use det(AB) = (det A) (det B) to save time and reduce mostakes...

**3.** Find det(A), det(B) and det(AB) where

$$A = \begin{bmatrix} 1 & 2 & 17 \\ 0 & -2 & 8 \\ 0 & 0 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\beta = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Gamma_1 \leftarrow 2\Gamma_1$$

(1 & 2 C) (2 & 2 C) (4 & 2 C) **4.** Consider the matrix A with reduced row eschelon form R where

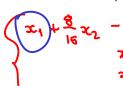


$$A = \begin{bmatrix} \frac{5}{2} & \frac{4}{3} & -\frac{5}{2} & 0 \\ -\frac{5}{4} & -\frac{2}{3} & \frac{5}{4} & \frac{4}{3} \\ -\frac{15}{2} & -4 & \frac{15}{2} & \frac{4}{9} & \frac{4}{2} \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{7}{4} & \frac{7}{4} & \frac{7}{4} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & \frac{7}{4} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & \frac{7}{4} \end{bmatrix}$$

(i) Find a basis for  $Col(A) = \{Az : x \in \mathbb{R}^5 \}$ 

$$\left\{ \begin{bmatrix} 5h \\ -5/4 \\ -15/h \end{bmatrix}, \begin{bmatrix} 0 \\ 9/3 \\ 1/9 \end{bmatrix}, \begin{bmatrix} -2/3 \\ -7/b \\ 1/2 \end{bmatrix} \right\}$$

(ii) Find a basis for 
$$Nul(A) \approx \{x: Ax=0\}$$
 =  $\{x: Rx=0\}$ 



$$=\begin{bmatrix} -\frac{q}{16} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3$$
basis

**5.** Let A be the matrix and x be the vector given by

$$A = \begin{bmatrix} 2 & 6 & 7 \\ 3 & 2 & 7 \\ 5 & 6 & 4 \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.$$

Show that x is an eigenvector of A and find the eigenvalue.

- **6.** Answer the following true false questions:
  - (i) Whenever a system has free variables, the solution set contains a unique solution.
    - (A) True
    - (B) False
  - (ii) An inconsistent system has more than one solution.
    - (A) True
    - (B) False
  - (iii) When two linear transformations are performed one after another, the combined effect may not always be a linear transformation.
    - (A)True
    - (B) False
  - (iv)  $\det(A^{-1}) = 1/\det(A)$ .
    - (A) True
    - (B) False
  - (v) Cramer's rule can only be used for invertible matrices.
    - (A) True
    - (B) False
  - (vi) If W is a subspace of  $\mathbb{R}^n$  and v is in both W and  $W^{\perp}$ , then v=0.
    - (A) True
    - (B) False
  - (vii) If A = QR where Q has orthonormal columns, then  $R = Q^T A$ .
    - (A) True
    - False
- (viii) If  $A \in \mathbb{R}^{n \times n}$  is symmetric, there exists an orthonormal basis of  $\mathbb{R}^n$  which consists of eigenvectors of A.
- (A) True (B) False (ix) Every matrix  $A \in \mathbf{R}^{n \times n}$  can be factored as  $A = SDS^{-1}$  where D is diagonal and S is an invertible matrix.

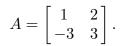
7. Suppose  $A \in \mathbf{R}^{2\times 3}$  is given by



$$A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 5 & 1 \end{bmatrix}.$$

How many free variables does the equation Ax=0 have? Find all solutions to the equation Ax=0.

**8.** Suppose  $A \in \mathbf{R}^{2 \times 2}$  is given by



Use the Gram-Schmidt algorithm to factor A=QR where Q is a matrix with orthonormal columns and R is upper triangular.



**9.** Find the eigenvalues and eigenvectors of the matrix A where

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}.$$



10. The LU factorization of a matrix A is given by

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/3 & -2/3 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{bmatrix}.$$

Explain how to use this factorization to solve the equation Ax = b and then find the value of x corresponding to b = (4, 6, 17).

11. The QR factorization of a matrix A is given by

ath 330: Final Exam Version A Sample Final . The 
$$QR$$
 factorization of a matrix  $A$  is given by 
$$Q = \begin{bmatrix} \frac{1}{3} & \frac{2}{3\sqrt{5}} \\ -\frac{2}{3} & \frac{\sqrt{5}}{3} \\ \frac{2}{3} & \frac{4}{3\sqrt{5}} \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 3 & \frac{1}{3} \\ 0 & \frac{4\sqrt{5}}{3} \end{bmatrix}.$$

Explain how to use this factorization to minimize ||Ax - b|| and then find the minimizing value of x corresponding to b = (1, 0, 1).

# 12. The matrix A given by



$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

has eigenvalues  $\lambda_i$  and eigenvectors  $x_i$  given by

$$\lambda_1 = 2, \quad x_1 = \begin{bmatrix} 2/3 \\ 1 \\ 1 \end{bmatrix}, \qquad \lambda_2 = 3, \quad x_2 = \begin{bmatrix} 1/4 \\ 3/4 \\ 1 \end{bmatrix}, \qquad \lambda_3 = 1, \quad x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Find an invertible matrix S and a diagonal matrix D such that  $A = SDS^{-1}$ .

(i) What is D?

(ii) What is S?