- The simplications that come with diagonal matrices and the factorization $A = SDS^{-1}$ only work if A has eigenbasis.

- Note the matrix $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ has only one linearly independent eigenvector and therefore no eigenbasis.

Something about eigenvalues and eigenvectors...

- The spectral Theorem:

If A is symmetric then it has an orthonormal eigenbasis.

Note then $S = \begin{bmatrix} x_1 \,|\, x_2 \,|\, \cdots \,|\, x_n \end{bmatrix}$

orthonormal columns

Means $S^T S = I$ since S is square this further implies $S^{-1} = S^T$.

called an orthogonal matrix

Therefore if $A = A^T$ then $A = SDS^T$

since $S^T = S^{-1}$ this is still a similarity transformation...

We study the eigenvalue eigenvector problem when A is symmetric...

Main thing I can do with symmetric matrices is move them through a dot product...

$$x \cdot Ay = x^T Ay = x^T A^T y = (Ax)^T y = Ax \cdot y$$

The equation $x \cdot Ay = Ax \cdot y$ for all vectors x and y could be taken as the definition of a symmetric matrix

Suppose $Ax = \lambda x$ and $A = A^T$.

$$Ax \cdot x = \lambda x \cdot x$$

Claim λ must be a real number... this isn't obvious, because λ is obtained by solving

$$\underbrace{\det(A - \lambda I) = 0}_{\text{polynomial equation...}}$$