Discussion of what the solution of

$$Rx = Q^T b$$

means when $Ax=b$ was inconsistent and didn't have a solution.

Recall $A = QR$

Claim: $\text{Col}\, A = \text{Col}\, Q$ since R is invertible...

Suppose $\boxed{M = BC}$ then $\text{Col}\, M \subseteq \text{Col}\, B$

$\quad\; m{\times}n \quad m{\times}k \quad k{\times}n$

$\text{Col}\, B = \{Bz : z \in \mathbb{R}^k\} \subseteq \mathbb{R}^m$

$\text{Col}\, C = \{Cy : y \in \mathbb{R}^n\} \subseteq \mathbb{R}^k$

$z \in \text{Col}\, C$ means $z = Cy$ for some $y \in \mathbb{R}^n$

$\text{Col}\, M = \{Mx : x \in \mathbb{R}^n\} \subseteq \mathbb{R}^m$

$\text{Col}\, M = \{Mx : x \in \mathbb{R}^n\} = \{B(Cx) : x \in \mathbb{R}^n\}$

$z = Cx$

$= \{Bz : \underbrace{z = Cx \text{ and } x \in \mathbb{R}^n}_{\text{just a substitution}}\} = \{Bz : z \in \text{Col}\, C\}$

$\subseteq \{Bz : z \in \mathbb{R}^k\}$ since $\text{Col}\, C \subseteq \mathbb{R}^k$

Therefore $\text{Col}\, M \subseteq \text{Col}\, B$. / or $\text{Col}\, BC \subseteq \text{Col}\, B$

Recall $A = QR$

<u>Claim</u>: $\text{Col } A = \text{Col } Q$ since R is invertible...

by the previous page $\text{Col } A \subseteq \text{Col } Q$

Since R is invertible then

$$A = QR$$

$$AR^{-1} = Q(RR^{-1})$$

$$AR^{-1} = QI$$

$$Q = AR^{-1}$$

by the previous page with this $\text{Col } Q \subseteq \text{Col } A$

$$\text{Col } A = \text{Col } Q$$

The $\text{Col } Q$ is easy to understand because the columns of Q are orthonormal, that is, unit vectors which are perpendicular to each other.

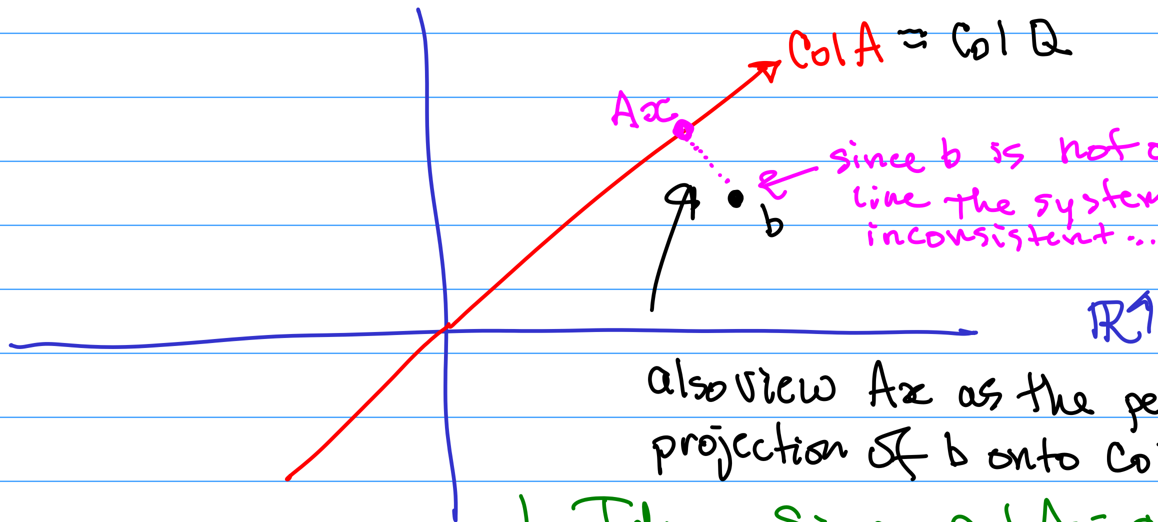
Let's go back to discussing

$$Ax = b$$ ★ overdetermined system
(4 equations
3 unknowns)

If $b \in \text{Col } A$ the system is consistent

If $b \notin \text{Col } A$ the system is inconsistent.

Inconsistent example (cartoon)



since b is not on the red line the system is inconsistent...

also view Ax as the perpendicular projection of b onto Col A.

Idea: Since Col A = Col Q then we can use the fact that Q has orthonormal columns to find the perpendicular projection easily — next time