row $\downarrow$ cols $\swarrow$

$\mathbb{R}^{m\times n}$

EM 3

Let A be an $m \times n$ matrix. The orthogonal complement of the row space of A is the null space of A, and the orthogonal complement of the column space of A is the null space of A^T:

$$(\text{Row } A)^\perp = \text{Nul } A \quad \text{and} \quad (\text{Col } A)^\perp = \text{Nul } A^T$$

$$\text{Col } A = \{ Ax : x \in \mathbb{R}^n \} \subseteq \mathbb{R}^m$$

$$\text{Nul } A = \{ x : Ax = 0 \} \subseteq \mathbb{R}^n$$

- dim Col A = # of pivots in the row eschelon form of A
- dim Nul A = # of free variables in the row eschelon form of A

Consequence

$$\underbrace{\dim \text{Col } A + \dim \text{Nul } A = n}$$

immediate consequence

The Invertible Matrix Theorem (continue

Let A be an $n \times n$ matrix. Then the follow the statement that A is an invertible matrix.

m. The columns of A form a basis of $\mathbb{R}^n$.

n. $\text{Col } A = \mathbb{R}^n$

o. $\dim \text{Col } A = n$

p. $\text{rank } A = n$

q. $\text{Nul } A = \{\mathbf{0}\}$

r. $\dim \text{Nul } A = 0$

$A \in \mathbb{R}^{m\times n}$

Row space... what section 4.2 page 215

$$A = \begin{bmatrix} r_1^T \\ r_2^T \\ \vdots \\ r_m^T \end{bmatrix}$$

$$\text{row } A = \text{Span}\{ r_1, r_2, \ldots, r_m \}$$

$$= \{ c_1 r_1 + c_2 r_2 + \cdots + c_m r_m : c_i \in \mathbb{R} \}$$

Connect Row A to some sort of columnspan

rows, cols

$$A^T = \begin{bmatrix} r_1 \,|\, r_2 \,|\, \cdots \,|\, r_m \end{bmatrix} \in \mathbb{R}^{n\times m}$$

The same...

$$\text{Col } A^T = \{ A^T c : c \in \mathbb{R}^m \} = \{ c_1 r_1 + c_2 r_2 + \dots + c_m r_m : c \in \mathbb{R}^m \}$$

Therefore $\text{rows } A = \text{Col } A^T$

EM 3

Let A be an $m \times n$ matrix. The orthogonal complement of the row space of A is the null space of A, and the orthogonal complement of the column space of A is the null space of A^T:

$$(\text{Row } A)^\perp = \text{Nul } A \quad \text{and} \quad (\text{Col } A)^\perp = \text{Nul } A^T$$

is actually the same thing as written twice once you know that row A = Col A^T

$$(\text{Col } A^T)^\perp = \text{Nul } A$$

Let $B = A^T$ $\quad B^T = A$

$$\boxed{(\text{Col } B)^\perp = \text{Nul } B^T}$$

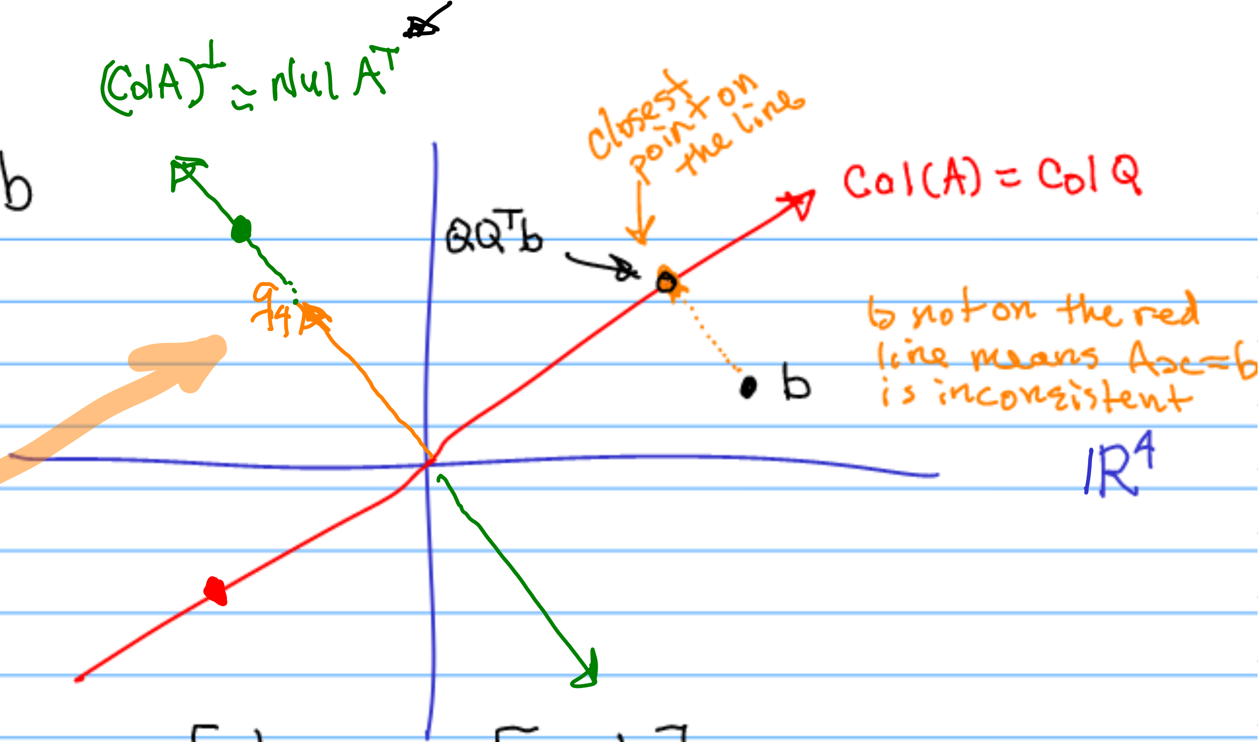
$(\text{Col } A)^\perp$

This is subspace that's perpendicular to the col A

If H is a subspace then

$$H^\perp = \{ x : x \cdot y = 0 \text{ for all } y \in H \}$$

$Ax=b$



$$QQ^Tb = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}\begin{bmatrix} q_1\cdot b \\ q_2\cdot b \\ q_3\cdot b \end{bmatrix} = (q_1\cdot b)q_1 + (q_2\cdot b)q_2 + (q_3\cdot b)q_3$$

If I put a q_4 here then what?