Connection between linear function and matrix vector multiplication...

Simple example $\quad f : \mathbb{R}^3 \to \mathbb{R}^2$

linear function

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 + 4x_3 \\ -x_1 + 2x_2 - x_3 \end{bmatrix}$$

a function that satisfies

① $\quad f(x+y) = f(x) + f(y)$

② $\quad f(\alpha x) = \alpha f(x)$

is a linear function...

Note that $\quad f(x) = Ax$

This is a 2×3 matrix

where
$$A = \begin{array}{cc} & \text{3 cols} \\ \text{2 rows} & \begin{bmatrix} 2 & -1 & 4 \\ -1 & 2 & -1 \end{bmatrix} \end{array} \in \mathbb{R}^{2\times 3} \quad \text{and} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Any function that satisfies ① & ② can be written as $f(x) = Ax$.

Let $f : \mathbb{R}^n \to \mathbb{R}^m$ that's linear...

find the matrix A...

Let $x \in \mathbb{R}^n$ then $\left(e_1 = \begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}, e_2 = \begin{bmatrix}0\\1\\0\\\vdots\\0\end{bmatrix}, \dots \right)$

$$x = \begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix} = x_1 \underbrace{\begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}}_{e_1} + x_2 \underbrace{\begin{bmatrix}0\\1\\0\\\vdots\\0\end{bmatrix}}_{e_2} + \cdots + x_n \underbrace{\begin{bmatrix}0\\\vdots\\0\\1\end{bmatrix}}_{e_n}$$

$$x = x_1 e_1 + x_2 e_2 + \dots + x_n e_n$$

$$f(x) = \underbrace{x_1 f(e_1) + x_2 f(e_2) + \dots + x_n f(e_n)}$$

So to find $f(x)$ it's only necessary to know the values of $f(e_1), f(e_2), \dots, f(e_n)$.

range

$$\boxed{f : \mathbb{R}^n \to \mathbb{R}^m}$$

Note that is matrix-vector mult. (by definition)

What are dimensions of A in general?

$$\begin{array}{cc} & n\text{-cols} \\ A = m \text{ rows} & \left[\begin{array}{c|c|c|c} f(e_1) & f(e_2) & \dots & f(e_n) \end{array}\right] \in \mathbb{R}^{m \times n} \end{array} \qquad x = \begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix}$$

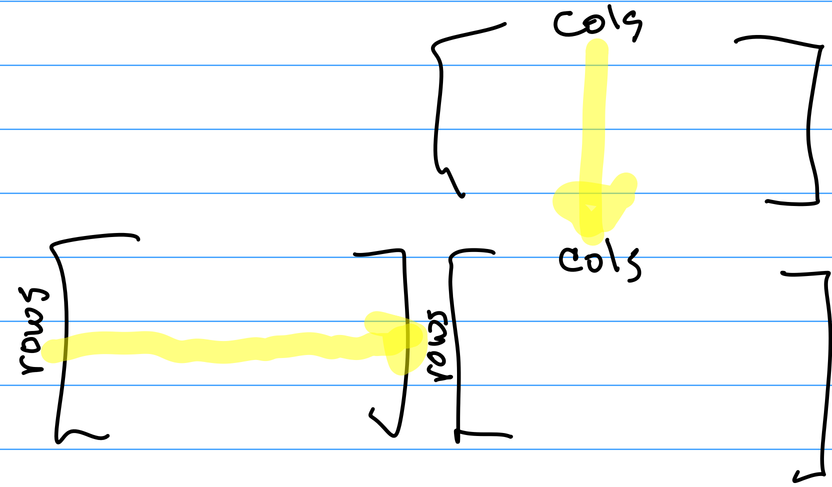
Then

$$Ax = \left[\begin{array}{c|c|c|c} f(e_1) & f(e_2) & \dots & f(e_n) \end{array}\right] \begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix}$$

$= f(e_1)x_1 + f(e_2)x_2 + \cdots + f(e_n)x_n = f(x)$

Chapter 2.

2.1 Matrix-Matrix Multiplication



2.2 Inverse Matrices...

If A is the matrix for the linear func.
$f(x) = Ax$.

Then A^{-1} should be the matrix for the (lin). funct
$g(x)$ such that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.

Do $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$
mean the same thing?

Simple example from regular algebra II involving $f(x) = \sqrt{x}$ and $g(x) = x^2$.

Note there are not linear functions
But serve as an example of what can go wrong with inverses...

• domain of x^2 is all real numbers

$$(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2} = |x|$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 = x$$

domain of $\sqrt{x}$ is $x \in [0, \infty)$

Same problem with linear functions...

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

Suppose I wanted $(f \circ g)(x) = x$ what about g?

$\mathbb{R}^2 \xrightarrow{g} \mathbb{R}^3 \xrightarrow{f} \mathbb{R}^2$ then $g : \mathbb{R}^2 \to \mathbb{R}^3$

Suppose I wanted $(g \circ f)(x) = x$ what about g?

$\mathbb{R}^3 \xrightarrow{f} \mathbb{R}^2 \xrightarrow{g} \mathbb{R}^3$ again $g : \mathbb{R}^2 \to \mathbb{R}^3$