LU - Matrix factorization

Idea given a matrix A find factors L and U such that $A = LU$.

lower triangular

upper triangular ← row eschelon form...

11. $\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix} = A$.

Make the row eschelon form using elimination steps...

$r_2 \leftarrow r_2 - \alpha r_1$

$r_3 \leftarrow r_3 - \beta r_1$

$\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$

$r_2 \leftarrow r_2 - 2r_1$ (row, col) = (2,1)

$r_3 \leftarrow r_3 - \left(-\frac{1}{3}\right) r_1$ ~~=~~ = (3,1)

$\begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 5 & 1 \end{bmatrix}$

$r_3 \leftarrow r_3 - 1r_2$ (row, col) = (3,2)

$$\begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix} = U$$

What is L?

$$L = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix} \end{array}$$

always the same...

Check the answer...

Multiply LU and see what we get...

11. $\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$

$$U = \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix}$$

usually if something goes wrong you can tell from this last entry...

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & \boxed{0} \end{bmatrix} = LU$$

What to do with LU factorization...

Solve $Ax = b$...

$LUx = b$

Substitute $y = Ux$

$$\left.\right\} \boxed{\begin{matrix} Ly = b \\ Ux = y \end{matrix}}$$

two simpler linear alg. problems...