

Chapter 2.5 : LU Matrix factorization...

Given a matrix A find a Lower triangular matrix L and an upper triangular Matrix U such that $LU = A$.

↑
Just a different name for the row echelon form...

Explain by means of an example...

$$12. \begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix}$$

Find the row echelon form of A ... This is done only using elimination steps

$$\left. \begin{array}{l} r_2 \leftarrow r_2 - \alpha r_1 \\ r_3 \leftarrow r_3 - \beta r_1 \\ \vdots \end{array} \right\}$$

only these, otherwise there is too much to keep track of...

$$12. \begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix}$$

$$r_2 \leftarrow r_2 - \frac{1}{2}r_1 \quad (\text{row, col}) = (2, 1)$$

$$r_3 \leftarrow r_3 - (-3)r_1 \quad (\text{row, col}) = (3, 1)$$

really $r_3 + 3r_1$

$$\begin{bmatrix} 2 & -4 & 2 \\ 0 & 7 & -5 \\ 0 & -14 & 10 \end{bmatrix}$$

$$r_3 \leftarrow r_3 - (-2)r_2 \quad (\text{row, col}) = (3, 2)$$

$$\begin{bmatrix} 2 & -4 & 2 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{bmatrix} = U$$

row echelon form
What is L?

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ -3 & -2 & 1 \end{bmatrix}$$

red part is always the same.

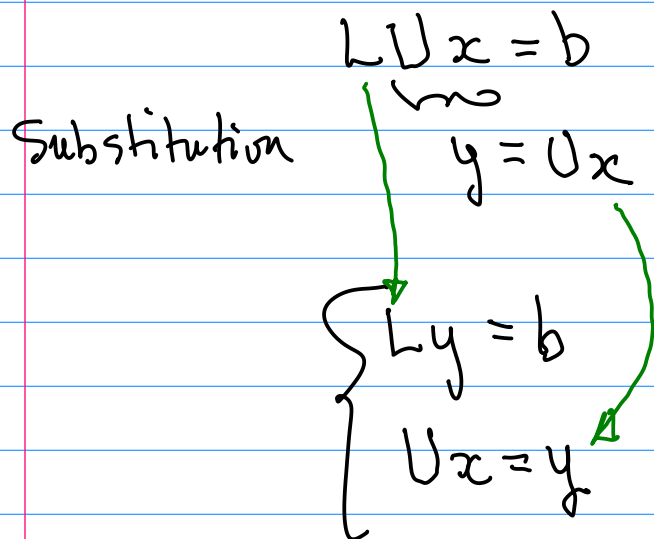
$$12. \begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & -4 & 2 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ -3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix} = LU$$

What is LU factorization good for?

Solve $Ax = b$, since $A = LU$



Solve 2 easier problems
instead of one
difficult problem...