Chapter 2.5 : LU Matrix factorization...

Given a matrix A find a Lower triangular matrix L and an upper triangular Matrix U such that $LU = A$.

Just a different name for the row eschelon form...

Explain by means of an example...

12. $\begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix}$

Find the row eschelon form of A... This is done only using elimination steps

$$\left.\begin{aligned} r_2 &\Leftarrow r_2 - \alpha r_1 \\ r_3 &\Leftarrow r_3 - \beta r_1 \\ &\vdots \end{aligned}\right\}$$

only these, otherwise there is too much to keep track of...

12. $\begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix}$

$r_2 \leftarrow r_2 - \frac{1}{2} r_1$ (row, col) = (2, 1)

$r_3 \leftarrow r_3 - (-3) r_1$ (row, col) = (3, 1)

really $r_3 + 3r_1$

$$\begin{bmatrix} 2 & -4 & 2 \\ 0 & \boxed{7} & -5 \\ 0 & -14 & 10 \end{bmatrix}$$

$r_3 \leftarrow r_3 - (-2) r_2$ (row, col) = (3, 2)

$$\begin{bmatrix} 2 & -4 & 2 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{bmatrix} = U$$

row eschelon form

What is L?

$$L = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -3 & -2 & 1 \end{bmatrix} \end{array}$$

← red part is always the same.

---

12. $\begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix}$

$$U = \begin{bmatrix} 2 & -4 & 2 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix} = LU.$$

What is LU factorization good for?

Solve $Ax=b$, since $A=LU$

$$L\underbrace{Ux}_{y=Ux}=b$$

Substitution

$$\begin{cases} Ly=b \\ Ux=y \end{cases}$$

Solve 2 easier problems instead of one difficult problem...