Sept 17, 2001

$$U = \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix}$$

usually something wrong can: the

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & \boxed{0} \end{bmatrix} = LU$$

Given

$$A = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

How does LU help solve $Ax = b$?

$$\underbrace{LUx}_{y} = b$$

$$\begin{cases} Ly = b \\ Ux = y \end{cases}$$

$-\frac{1}{3}y_1 + y_2 + y_3 = 3$

$y_3 = 3 + \frac{1}{3}y_1 - y_2 = 3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$y_1 = 1$

$y_2 = 2 - 2y_1 = 2 - 2 = 0$

$y_3 = \cancel{13/3}$ or $\boxed{10/3}$

$$\begin{bmatrix} 3 & -6 & 3 \\ 0 & \boxed{5} & -4 \\ 0 & 0 & \boxed{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 10/3 \end{bmatrix}$$

$x_1 = \quad 11/15$

$x_2 = \frac{1}{5}(0 + 4x_3) = 8/15$

$x_3 = 2/3$

$$3x_1 - 6x_2 + 3x_3 = 1$$

$$3x_1 = 1 + 6x_2 - 3x_3$$

$$x_1 = \frac{1}{3}\left(1 + 6\cdot\frac{8}{15} - 3\cdot\frac{2}{3}\right) = \boxed{}$$

Solution to $Ax=b$ is $x = \begin{bmatrix} 11/15 \\ 8/15 \\ 2/3 \end{bmatrix}$

Chapter 2.8 ...

DEFINITION

A **subspace** of $\mathbb{R}^n$ is any set H in $\mathbb{R}^n$ that has three properties:

a. The zero vector is in H.

b. For each $\mathbf{u}$ and $\mathbf{v}$ in H, the sum $\mathbf{u} + \mathbf{v}$ is in H.

c. For each $\mathbf{u}$ in H and each scalar c, the vector $c\mathbf{u}$ is in H.

Different example: Set of solutions to the equation $Ax=0$ is a subspace. (when there are free variables in the problem)

(Called Null space of A)