Catch up: Elementary row operation

1. Elimination step
2. row swap
3. rescaling

① $r_i \leftarrow r_i - \alpha r_j$

This can be seen as a linear function..

Example: $f(A)$ = what you get after performing $r_2 \leftarrow r_2 + 2r_1$

$(\alpha = -2)$

Need to check:

$$\begin{cases} ① \ f(A+B) = f(A) + f(B) \\ ② \ f(mA) = m f(A) \end{cases}$$

Not proving anything, just testing the idea.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$r_2 \leftarrow r_2 + 2r_1$ $\qquad$ $r_2 \leftarrow r_2 + 2r_1$

$$f(A) + f(B) = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 9 & 12 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 4 & 1 & 4 \\ 0 & 3 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & 2 & 4 \\ 10 & 10 & 16 \\ 7 & 11 & 13 \end{bmatrix}}$$

$$A+B = \begin{bmatrix} 2 & 2 & 4 \\ 6 & 6 & 8 \\ 7 & 11 & 13 \end{bmatrix} \quad r_2 \leftarrow r_2 + 2r_1 \quad \boxed{\begin{bmatrix} 2 & 2 & 4 \\ 10 & 10 & 16 \\ 7 & 11 & 13 \end{bmatrix}}$$

Note $\quad mf(A) = f(mA)$ also works just fine, but won't check this...

If f is a linear function then there is a matrix that corresponds to it...

Recall connection between matrix-matrix mult and matrix-vector mult.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \left[\begin{array}{c|c|c} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 0 & 3 & 4 \end{array}\right]$$

$$AB = \left[\begin{array}{c|c|c} A\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} & A\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} & A\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \end{array}\right]$$

The matrix for f is given by $f(x) = Mx$

$$M = \left[f(e_1) \,\middle|\, f(e_2) \,\middle|\, \cdots \,\middle|\, f(e_n) \right] =$$

$$= f\left(\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ 0 & 0 & & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & \vdots & & 1 \end{bmatrix} \right) = f(I)$$

Example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad r_2 \leftarrow r_2 + 2r_1 \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$r_2 \leftarrow r_2 + 2r_1$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 6 & 9 & 12 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ 6 & & \\ 7 & & \end{bmatrix}$$

② Try $r_2 \leftrightarrow r_3$ (swap rows)

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad r_2 \leftrightarrow r_3 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = M$$

③ Rescaling

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad r_2 \leftarrow \frac{1}{2} r_2 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = M$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5/2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

Notation

$$\left[r_2 \leftarrow \frac{1}{2} r_2 \right] \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

① Let $A \in \mathbb{R}^{4 \times 6}$. Find the matrix M corresponding to the row operation $r_3 \leftarrow r_3 - r_2$ as applied to A.

Gave A but don't really need to know A...

All I need are the dimensions of A.

Actually only the 4.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad r_3 \leftarrow r_3 - r_2 \quad M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$[r_3 \leftarrow r_3 - r_2]$