Required Texts:

Linear Algebra and Its Applications, n-th Edition by David C. Lay.

https://www.pearson.com/mylab (class registration code)

click here from the course webpage

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$$\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 5x_1 - 5x_3 = 10 \end{cases}$$

Solve elimination and substitution... } If you've done this before...

First time

$$\begin{cases} x - 2y + z = 0 \\ 2y - 8z = 8 \\ 5x - 5z = 10 \end{cases}$$

already using subscripts to denote different variables...

vectors

column vector

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ mean same thing...

or $x = (x_1, x_2, x_3)$

still a column vector written word...

$\boxed{x \in \mathbb{R}^3}$ means

three-dimensional vector with real-number entries

also

$x \in \mathbb{R}^n$

a vector with number of components indicated by the parameter n

Generalized notation

$x \in \mathbb{C}^3$

three-dimensional vector with complex-number entries

$$\begin{cases} 1x_1 - 2x_2 + x_3 = 0 \\ \quad\;\; 2x_2 - 8x_3 = 8 \\ 5x_1 \quad\;\; - 5x_3 = 10 \end{cases}$$

$n = 3$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$$

Matrix of coefficients

another vector

$$A = \begin{bmatrix} 1 & -2 & +1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 8 \\ 10 \end{bmatrix}$$

All the inputs and outputs to linear algebra problem have been encoded into vectors and a matrix.

The same problem is written as $Ax = b$

The method of elimination and substitution for solving Ax=b can be reinterpreted as a way of factoring the matrix A into two simpler matrices L and U.

What does it mean for A = LU ?

Left hand side of the linear system

$$\begin{matrix} 1x_1 - 2x_2 + x_3 \\ 2x_2 - 8x_3 \\ 5x_1 \quad - 5x_3 \end{matrix}$$

three linear functions

or one vector-valued function

$$f(x) = \begin{bmatrix} 1x_1 - 2x_2 + x_3 \\ 2x_2 - 8x_3 \\ 5x_1 - 5x_3 \end{bmatrix}$$

$\boxed{f(x) = Ax}$ left hand side of the linear system in matrix notation

$f: \mathbb{R}^3 \to \mathbb{R}^3$ vector valued function of a vector input

In calculus

column vector...

$$f(x_1, x_2, x_3) = (x_1 - 2x_2 + x_3,\ 2x_2 - 8x_3,\ 5x_1 - 5x_3)$$

Consider another linear function $g: \mathbb{R}^3 \to \mathbb{R}^3$

$$g(x) = \begin{bmatrix} x_2 - 3x_3 \\ 2x_1 + 4x_2 \\ x_1 + x_2 + x_3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & -3 \\ 2 & 4 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad g(x) = Bx$$

Algebraically one can add, substract and compose vector-valued functions...

$$(f+g)(x) = \begin{bmatrix} 1x_1 - 2x_2 + x_3 \\ 2x_2 - 8x_3 \\ 5x_1 - 5x_3 \end{bmatrix} + \begin{bmatrix} x_2 - 3x_3 \\ 2x_1 + 4x_2 \\ x_1 + x_2 + x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 - 2x_3 \\ 2x_1 + 6x_2 - 8x_3 \\ 6x_1 + x_2 - 4x_3 \end{bmatrix}$$

$$Ax + Bx = \begin{bmatrix} x_1 - x_2 - 2x_3 \\ 2x_1 + 6x_2 - 8x_3 \\ 6x_1 + x_2 - 4x_3 \end{bmatrix} \qquad A + B = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 6 & -8 \\ 6 & 1 & -4 \end{bmatrix}$$

Matrix addition is the matrix you get from adding the corresponding linear functions.

Matrix multiplication is the matrix you get from composing the corresponding linear functions...