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Required Texts:

Linear Algebra and Its Applications, n-th Edition by David C. Lay.
https://www.pearson.com/mylab (class registration code)

Other resources:

MIT Open Courseware, Gilbert Strang, Spring 2010.
18-06-linear-algebra-spring-2010

Introduction to Applied Linear Algebra, Boyd and Vandenberghe.
http://vmls-book.stanford.edu/

if not registered online for the publisher's homework please do over the weekend...

Lecture Notes

- Lecture 1: Overview of the Course
- Lecture 2: Systems of Linear Equations

Matrices represent linear functions. What can you do with functions?

. Add and subtract them.

. Compose them

. Find their inverses.

. Factor them, that is write f(x)=g(h(x)) the composition of two other functions.

$$f(x) = \begin{bmatrix} 1x_1 - 2x_2 + x_3 \\ 2x_2 - 8x_3 \\ 5x_1 - 5x_3 \end{bmatrix}$$

$\mathbb{R}^3$

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ A = & \multicolumn{3}{c}{\begin{bmatrix} 1 & -2 & +1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix}} \end{array}$$

$$g(x) = \begin{bmatrix} x_2 - 3x_3 \\ 2x_1 + 4x_2 \\ x_1 + x_2 + x_3 \end{bmatrix}$$

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ B = & \multicolumn{3}{c}{\begin{bmatrix} 0 & 1 & -3 \\ 2 & 4 & 0 \\ 1 & 1 & 1 \end{bmatrix}} \end{array}$$

Note: not any function can be represented by a matrix. But only linear functions.
And any function which can be represented by a matrix is a linear function.

What is the matrix for the function $(f+g)(x)$?

$$\underline{A+B} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -3 \\ 2 & 4 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 6 & -8 \\ 6 & 1 & -4 \end{bmatrix}$$

Same as last time...

$$A+B = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 6 & -8 \\ 6 & 1 & -4 \end{bmatrix}$$

try to think about matrices, not as tables of numbers but as linear functions. Then all the operations we do start to make sense.

Note columns of matrix correspond to the inputs of the functions!

Note adding matrices is pretty much the same as adding weird shaped vectors...

Multiplication...is composition of the function...

What is the matrix of the function $(f \circ g)(x)$?

Closure property...

Note that...if you add two linear functions, you get another linear function...and if you compose two of them, you get another linear function...

means you can do these operations again and again $\rightarrow$ leads to an algebra of matrices..

$$f(x) = \begin{bmatrix} 1x_1 - 2x_2 + x_3 \\ 2x_2 - 8x_3 \\ 5x_1 - 5x_3 \end{bmatrix}$$

$\mathbb{R}^3$

$$g(x) = \begin{bmatrix} x_2 - 3x_3 \\ 2x_1 + 4x_2 \\ x_1 + x_2 + x_3 \end{bmatrix}$$

That substitutes for x_1 in definition of $f(x)$

for x_2

$$(f \circ g)(x) = f\left(\begin{bmatrix} x_2 - 3x_3 \\ 2x_1 + 4x_2 \\ x_1 + x_2 + x_3 \end{bmatrix}\right) = \begin{bmatrix} (x_2 - 3x_3) - 2(2x_1 + 4x_2) + (x_1 + x_2 + x_3) \\ 2(2x_1 + 4x_2) - 8(x_1 + x_2 + x_3) \\ 5(x_2 - 3x_3) - 5(x_1 + x_2 + x_3) \end{bmatrix}$$

simplify

$$= \begin{bmatrix} -3x_1 - 6x_2 - 2x_3 \\ -4x_1 + 0x_2 - 8x_3 \\ -5x_1 + 0x_2 - 20x_3 \end{bmatrix},$$

Thus,

$$AB = \begin{bmatrix} -3 & -6 & -2 \\ -4 & 0 & -8 \\ -5 & 0 & -20 \end{bmatrix}$$

Idea behind the whole course: Given a matrix can it be factored as a product of simpler matrices? Yes, but how?

- Gaussian Elimination finds the factorization A=LU ← matrices
- Gram-Schmidt algorithm finds the factorization A=QR ← q matrices
- Eigenvalue decomposition A= S D S^{-1}
- Singular value decomposition A = $U \Sigma V^T$ ← matrices...

q q — orthogonal matrix

diagonal...

main idea...the matrices that the original A is factored into are simpler in some way or the other...

Track how the coefficients combine to make $(f \circ g)(x)$

5) $AB = \begin{bmatrix} -3 & -6 & -2 \\ -4 & 0 & -8 \\ -5 & 0 & -20 \end{bmatrix}$

$$B = \begin{bmatrix} 0 & 1 & -3 \\ 2 & 4 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

← second matrix

Made out of dot products...

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix} \quad \begin{bmatrix} -3 & -6 & -2 \\ -4 & \# & \\ -5 & & \end{bmatrix}$$

over here first matrix

Answer AB

Note that the rows of matrix A are dotted into the columns of matrix B to get the product AB

The augmented matrix does not correspond to a linear function!

It's just a table of numbers...

$$\left[A \mid b \right]$$

← augmented matrix...

Matrix of coefficients

another vector

$$A = \begin{bmatrix} 1 & -2 & +1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 8 \\ 10 \end{bmatrix}$$

$$\begin{cases} 1x_1 - 2x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 5x_1 - 5x_3 = 10 \end{cases}$$

Augmented matrix

$$[A|b] = \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

The red line is optional...

The advantage of augmented matrix is that all the numbers for the linear algebra problem are included...

and when performing the Gaussian Elimination algorithm, it can be worked with as if it were a matrix...

Note:

Labor Day (no classes, campus closed) | Monday, 9/5/2022

We are in-person on Wednesday next week...