Definition of span...

If $\mathbf{v}_1, \dots, \mathbf{v}_p$ are in $\mathbb{R}^n$, then the set of all linear combinations of $\mathbf{v}_1, \dots, \mathbf{v}_p$ is denoted by Span $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ and is called the **subset of $\mathbb{R}^n$ spanned** (or **generated**) **by $\mathbf{v}_1, \dots, \mathbf{v}_p$**. That is, Span $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ is the collection of all vectors that can be written in the form

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_p\mathbf{v}_p$$

with $c_1, \dots, c_p$ scalars.

Theorem 5

If A is an $m \times n$ matrix, $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^n$, and c is a scalar, then:

a. $A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$;

b. $A(c\mathbf{u}) = c(A\mathbf{u})$.

} properties of a linear function...

From my point of view this is obvious, because

$f(x) = Ax$

was defined as the linear function of the left side of a system of linear equations...

Idea in 1.5 to solve when right side is 0.

Homogeneous Linear Systems

A system of linear equations is said to be **homogeneous** if it can be written in the form $A\mathbf{x} = \mathbf{0}$, where A is an $m \times n$ matrix and $\mathbf{0}$ is the zero vector in $\mathbb{R}^m$. Such a system

If interested in factoring $A = LU$ then don't need to worry whats on right... simplest set it equal 0.

when a computer solves $Ax=b$, the typical approach is to fact the left side first...

$$A = LU$$

Then we solve $\boxed{LUx=b}$. How?

Is that easy? yes...

introduce a new variable...

$$y = Ux$$

substitute

$$Ly = b$$

$$(f \circ g)(x) = b$$

$$f(g(x)) = b$$

$$y = g(x)$$

$$f(y) = b$$

same thing

Thus, in summary

$$\begin{cases} Ly = b \\ Ux = y \end{cases}$$

easy to solve for y here

plug in here

then solve for x... which is also easy...

Chapter 1.3 Dot product...

$x, y \in \mathbb{R}^4$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad y = \begin{bmatrix} -1 \\ -3 \\ 5 \\ 2 \end{bmatrix}$$

Then

$$x \cdot y = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -3 \\ 5 \\ 2 \end{bmatrix} = 1(-1) + (2)(-3) + (3)(5) + (4)(2)$$

$$= -1 - 6 + 15 + 8 = -7 + 23 = 16$$

Compare with Matrix multiplication

& transpose, switches rows with columns...

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

Note, we've been thinking about matrices as representing linear functions. THat means they are more than just tables of numbers...

What does it mean, then, to switch the rows with the columns?

Matrix product

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 5 \\ 2 \end{bmatrix}$$

What's the answer?

inner product between two vectors...

$$\begin{bmatrix} -1 \\ -3 \\ 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 16 \end{bmatrix}$$

The answer is here

What's the outer product? Transpose the other vector...

Transpose of other vector...

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} -1 & -3 & 5 & 2 \end{bmatrix} = ?$$

Compute it. What's the answer

Outer product of two vectors...

$$\begin{array}{cc} & \begin{bmatrix} -1 & -3 & 5 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} & \begin{bmatrix} -1 & -3 & 5 & 2 \\ -2 & -6 & 10 & 4 \\ -3 & -9 & 15 & 6 \\ -4 & -12 & 20 & 8 \end{bmatrix} \end{array}$$

The answer is a 4x4 matrix

From 1.5 #19

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3 \end{aligned}$$

Next time

From 1.5 # 7

In Exercises 7–12, describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where A is row equivalent to the given matrix.

7. $\begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}$ **8.** $\begin{bmatrix} 1 & -2 & -9 & 5 \\ 0 & 1 & 2 & -6 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}$$

solve $\quad Ax = 0$

augmented matrix

$$\underbrace{\left[\begin{array}{cccc|c} 1 & 3 & -3 & 7 & 0 \\ 0 & 1 & -4 & 5 & 0 \end{array}\right]}_{\text{Echelon form...}}$$

since always zero, no write it

Make it reduced Echelon form...

$r_1 \leftarrow r_1 - 3r_2$

$$\begin{array}{c} \begin{array}{cccc} P & P & F & F \end{array} \\ \begin{bmatrix} 1 & 0 & 9 & -5 \\ 0 & 1 & -4 & 5 \end{bmatrix} \end{array}$$

Solution

$$\begin{cases} x_1 = -9x_3 + 5x_4 \\ x_2 = 4x_3 - 5x_4 \end{cases}$$

Parametric form

$$x = \begin{bmatrix} -9 \\ 4 \end{bmatrix} x_3 + \begin{bmatrix} 5 \\ -5 \end{bmatrix} x_4$$

for any choice of x_3 and x_4.