$Ax=0$ meant the same as $Ux=0$

This means if there is a dependency relation between the columns of A, those same dependency relations hold for the columns of U.

Note any $x \neq 0$ that solves $Ax=0$ is a dependency relation between the columns of A.

original matrix

$$A = \begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & -3 & -6 \end{bmatrix} \xrightarrow{\text{row operations}} \begin{bmatrix} 1 & 3 & 1 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \approx U$$

Echelon form of A

$$\begin{array}{cccccc} \text{last column} & & \text{second column} & & \text{first column} \\ \begin{bmatrix} 1 \\ 2 \\ -6 \end{bmatrix} & = 2 & \begin{bmatrix} 3 \\ -9 \\ -3 \end{bmatrix} & -5 & \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix} \end{array}$$

$$\begin{array}{cccccc} \text{last column} & & \text{second column} & & \text{first column} \\ \begin{bmatrix} 1 \\ 6 \\ 0 \end{bmatrix} & = 2 & \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} & -5 & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{array}$$

From monday we solved $Ax=0$ and obtaind.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} x_3$$

Thus setting $x_3 = 1$ I get that $x = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$ solves $Ax=0$

anything I like for x_3

Since there were free variable then $Ax=0$ had non-trivial solutions.

$$\begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & -3 & -6 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} = 0$$

In the column representation of matrix multiplication that mean

$$\begin{bmatrix}1\\-4\\0\end{bmatrix}5 + \begin{bmatrix}3\\-9\\-3\end{bmatrix}(-2) + \begin{bmatrix}1\\2\\6\end{bmatrix}(1) = 0$$

Thus

$$\begin{bmatrix}1\\2\\6\end{bmatrix} = -\begin{bmatrix}1\\-4\\0\end{bmatrix}5 - \begin{bmatrix}3\\-9\\-3\end{bmatrix}(-2)$$

(Same dependency as found before)

If a matrix has more column than rows there must be at least 1 free variable.

If a set contains more vectors than there are entries in each vector, then the set is linearly dependent. That is, any set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ in $\mathbb{R}^n$ is linearly dependent if $p > n$.

p = how many vectors n = length of the vectors

$p = 4$ $n = 3$ $4 > 3$

$$\left\{ \begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}-1\\2\\7\end{bmatrix}, \begin{bmatrix}-2\\-3\\4\end{bmatrix} \right\}$$

put the vectors into a matrix

$$A = \begin{bmatrix}1 & 1 & -1 & -2\\2 & 0 & 2 & -3\\3 & 1 & 7 & 4\end{bmatrix}$$

- To find dependency relations in the columns of A we solve $Ax = 0$
- To solve $Ax = 0$ we find the Echelon form of A.

Now do row operations on A ...

elimination steps $r_i \leftarrow r_i - \alpha r_j$

to obtain U, the Echelon form...

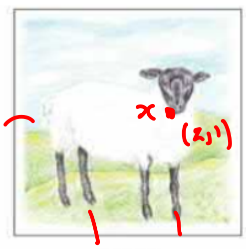
$$U = \begin{array}{c} \\ 3 \text{ rows} \end{array}\overset{4 \text{ columns}}{\begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}}$$

Ask question are there any free variables?
what are are the pivots?
At most how many pivots could there be?

since each pivot needs a row there are at most 3 pivots ... There are 4 variables because 4 columns
Thus one of them must be a free variable...

If a matrix contains a column of zeros then it has a free variable.

If a set $S = \{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ in $\mathbb{R}^n$ contains the zero vector, then the set is linearly dependent.



sheep

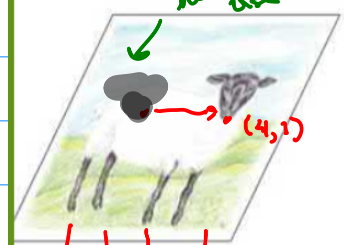
$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}.$$

row operation

$r_1 \leftarrow r_1 + 2r_2$

$$Ax = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

head of sheep used to be here



sheared sheep

check this

How to find the matrix corresponding to a row operation?
Perform the row operation on the identity matrix.

$$[r_1 \leftarrow r_1 + 2r_2] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

For next time, please read section 1.9 about the geometric interpretation of other linear transformations (functions).