a little more about transposes:

$$A \in \mathbb{R}^{m \times n} \qquad\qquad B \in \mathbb{R}^{n \times p}$$

Then AB makes since. Why because the dimensions line up.:

$$\underset{m \times n}{A} \; \underset{n \times p}{B} \in \mathbb{R}^{m \times p}$$

rows cols tells how many elements a row of A has...

To make this matrix involes lots of dot products between rows of A and columns of B

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & -1 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 3} \qquad B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & -5 \end{bmatrix} \in \mathbb{R}^{3\times 2}$$

Then $AB \in \mathbb{R}^{2\times 2}$

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & -5 \end{bmatrix}$$

answer is AB

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & -3 \\ 8 & -17 \end{bmatrix}$$

Same answer except transpose...

$$A^T = \begin{bmatrix} 1 & 0 \\ -2 & -1 \\ 0 & 3 \end{bmatrix}$$

answer is $(AB)^T$

$$B^T = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & -5 \end{bmatrix} \begin{bmatrix} 0 & 8 \\ -3 & -17 \end{bmatrix}$$

Conclusion $B^T A^T = (AB)^T$.

- The transpose of the product is the product of the transposes in reverse order.

When does switching columns and rows come up? in dot products...

since $u \cdot v = u^T v$

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

Then $u \cdot v = 4 + 10 + 18 = 32$

Also

$$u^T v = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 32$$

In the end matrix multiplication is related to collecting coeficients when composing linear funcs.

From last time I considered

$$Ax \cdot y = (Ax)^T y = (x^T A^T) y$$

associativity of Matrix products ... i.e. associativity of funct. comp.

$$= x^T (A^T y) = x \cdot A^T y$$

Thus $\underbrace{Ax \cdot y = x \cdot A^T y}_{\text{jump}}$

when A jumps over a dot product it gets a transpose

Note if $A^T = A$ then the matrix doesn't change when it jumps across a dot product... and that's called symmetric ...

If $A^T = A$ then A is $\boxed{\text{symmetric}}$.

We want to find A^{-1} ...

Section 2.2

Idea: solve $A \in \mathbb{R}^{n \times n}$

$$\underbrace{Av_1 = e_1, \quad Av_2 = e_2, \quad Av_n = e_n}$$

lots of systems of linear equations. same A on the left but different right sides

/

Solved these using Gaussian elimination to make the echelon form of A.

Reinterpreted this as a way to factor the matrix A into simpler matrices

mean triangular matrices

$$A = LU$$

upper triangular matrix (echelon form of A).

lower triangular matrix with 1's on the diagonal (we know how to find L from the coefficients in the elimination steps).

Once you know $A=LU$ then solving $Ax=b$ for different right sides is easy. How

$$Av_1 = e_1$$

$$LUv_1 = e_1$$

Solve these instead...

$$\begin{cases} Uv_1 = y \\ Ly = e_1 \end{cases}$$

Since U and L are triangular then solving can be done by substitution no more elimination is needed.

Thus solve all these systems:

$$\begin{cases} Uv_1 = y_1 \\ Ly_1 = e_1 \end{cases}, \quad \begin{matrix} Uv_2 = y_2 \\ Ly_2 = e_2 \end{matrix} \qquad \begin{matrix} Uv_n = y_n \\ Ly_n = e_n \end{matrix}$$

Directly doing the above is great if you're a computer, but difficult to organize on paper...
Based on augmented matrices...

Want to solve these...

$$Av_1 = e_1, \quad Av_2 = e_2, \qquad Av_n = e_n$$

Augmented matrix

$$\underbrace{\left[A \mid e_1\right] \quad \left[A \mid e_2\right], \cdots \left[A \mid e_n\right]}$$

do each one at a time would be awful... because the elimination steps are the same each time..

Use one really big augmented matrix

$$\left[\,A \;\middle|\; e_1 \;\middle|\; e_2 \;\middle|\; \cdots \;\middle|\; e_n\,\right]$$

now convert A to reduced echelon form...
and can read of the answers for $v_1, \ldots, v_n$ on the right

$$\left[\,I \;\middle|\; \underbrace{v_1 \;\middle|\; v_2 \;\middle|\; \cdots \;\middle|\; v_n}_{A^{-1}}\,\right]$$

we'll work an example w/numbers next time...