Example $n=3$

to find A^{-1} explicitly...

$$A^{-1} = \begin{bmatrix} \det A_1(e_1) & \det A_1(e_2) & \det A_1(e_3) \\ \det A_2(e_1) & \det A_2(e_2) & \det A_2(e_3) \\ \det A_3(e_1) & \det A_3(e_2) & \det A_3(e_3) \end{bmatrix} \Big/ \det A$$

almost done, but these determinants can be simplified...

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\det A_{\underset{\text{column}}{2}}(e_3) = \begin{array}{c} \begin{array}{ccc} & e_3 & \end{array} \\ \begin{bmatrix} a_{11} & 0 & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & 1 & a_{33} \end{bmatrix} \\ \begin{array}{ccc} & (3,2) & \end{array} \end{array} = (-1)^5 \det \begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix}$$

expand on 2^{nd} column.

$$\det A_2(e_3) = (-1)^{2+3} \det A_{32}$$

$\det A_i(\mathbf{e}_j) = (-1)^{i+j} \det A_{ji} = C_{ji}$

In general...

$$\det A_i(e_j) = \boxed{(-1)^{i+j} \det A_{ji}} = C_{ji}$$

called the cofactors...

$$A^{-1} = \begin{bmatrix} \det A_1(e_1) & \det A_1(e_2) & \det A_1(e_3) \\ \det A_2(e_1) & \det A_2(e_2) & \det A_2(e_3) \\ \det A_3(e_1) & \det A_3(e_2) & \det A_3(e_3) \end{bmatrix} \Big/ \det A$$

$$= \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \Big/ \det A$$

Note the indexing is strange here. Nothing can be done to fix it because of tradition...

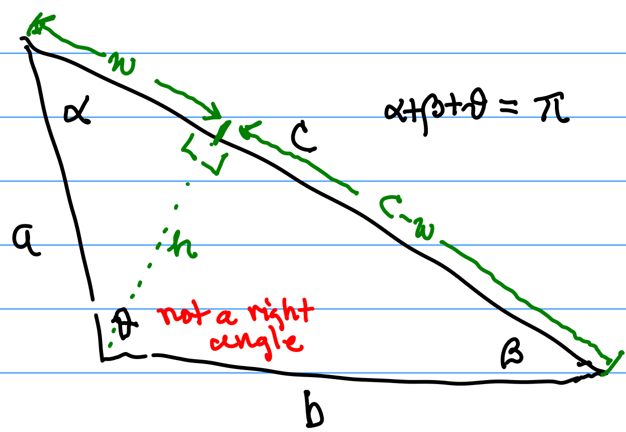
Chapter 3: Dot product again... application of orthogonality..

i.e. the dot product between two vectors is zero.

Law of cosines: In vector form is

$$\boxed{x \cdot y = \|x\| \, \|y\| \cos\theta}$$

where θ is the angle between x and y.

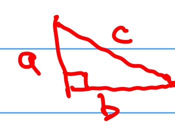


$\alpha + \beta + \theta = \pi$

Law of cosines is a generalization of the Pythagorean theorem to triangles that don't have a right angle.

recall Pythagorean theorem

$a^2 + b^2 = c$



$\sin^2\theta + \cos^2\theta = 1$

$\sin\alpha = \frac{h}{a}$ $\qquad$ $\sin\beta = \frac{h}{b}$

$\cos\alpha = \frac{w}{a}$ $\qquad$ $\cos\beta = \frac{c-w}{b}$

$w = a\cos\alpha$ $\qquad$ $c - w = b\cos\beta$

$$c^2 = ((c-w)+w)^2 = (c-w)^2 + 2(c-w)w + w^2$$

$$= b^2\cos^2\beta + 2b\cos\beta \, a\cos\alpha + a^2\cos^2\alpha$$

$$= b^2(1-\sin^2\beta) + 2ab\cos\beta\cos\alpha + a^2(1-\sin^2\alpha)$$

$$= \underbrace{a^2+b^2 \quad - a^2\sin^2\alpha + 2ab\cos\alpha\cos\beta - b^2\sin^2\beta}$$

simplify this term by substituting θ for the α and β terms.., use the angle addition formula and $\alpha+\beta+\theta=\pi$.

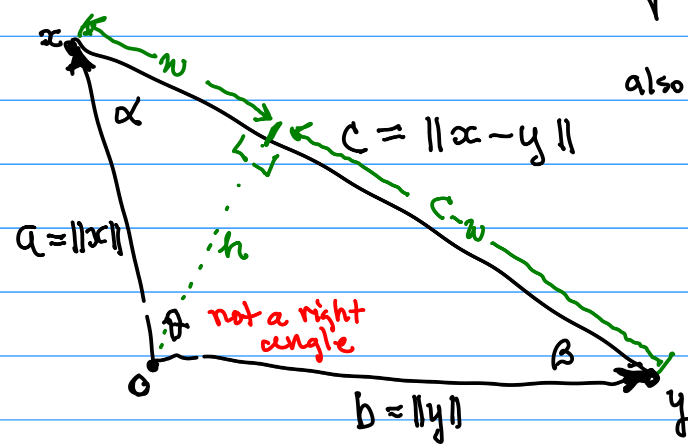
pretend I did that simplification

$$c^2 = a^2+b^2 - 2ab\cos\theta$$

Reinterpret the law of cosines in terms of vectors...

$\|x\| = \sqrt{x_1^2+x_2^2+\cdots+x_n^2}$ for $x \in \mathbb{R}^n$

also $\|x\| = \sqrt{x \cdot x}$



$$c^2 = a^2+b^2 - \boxed{2ab\cos\theta}$$

$$c^2 = \|x-y\|^2 = \left(\sqrt{(x-y)\cdot(x-y)}\right)^2 = (x-y)\cdot(x-y)$$

switch

$$= x\cdot x - x\cdot y - y\cdot x + y\cdot y = x\cdot x - 2x\cdot y + y\cdot y$$

compare these.

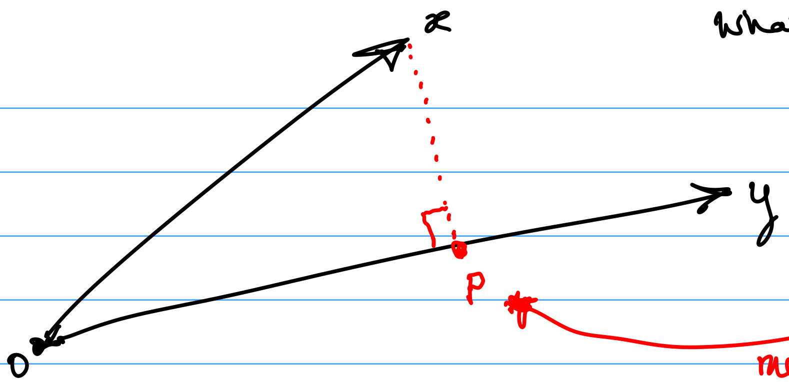
$$\boxed{\|x-y\|^2 = \|x\|^2 - 2x\cdot y + \|y\|^2}$$

$$c^2 = a^2+b^2 - \boxed{2x\cdot y}$$

Thus $2x\cdot y = 2ab\cos\theta$

$\boxed{x\cdot y = \|x\|\,\|y\|\cos\theta}$ where θ is the angle between x and y.

want to project x onto y.
What does it mean?



means find p.

we'll do this next time...