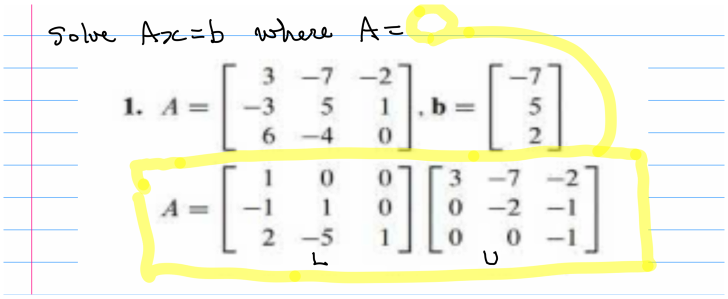
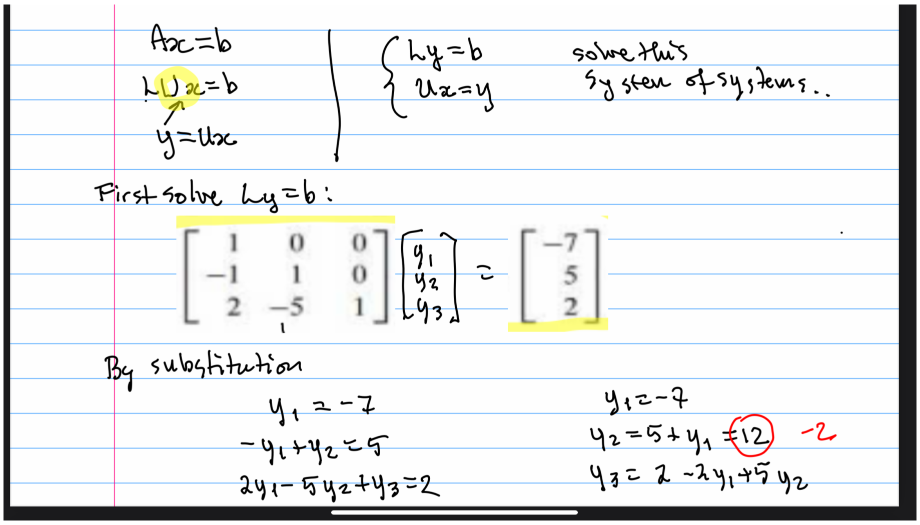


Solve $Ax=b$ where $A=$

1. $A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$

$$A = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix}}$$



$Ax=b$

$LUx=b$

$y=Ux$

$$\begin{cases} Ly=b \\ Ux=y \end{cases}$$

solve this system of systems..

First solve $Ly=b$:

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$$

By substitution

$y_1 = -7$

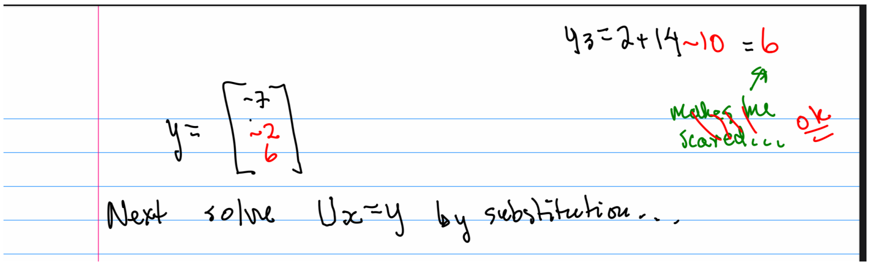
$-y_1 + y_2 = 5$

$2y_1 - 5y_2 + y_3 = 2$

$y_1 = -7$

$y_2 = 5 + y_1 = \boxed{12}$ -2

$y_3 = 2 - 2y_1 + 5y_2$



$y_3 = 2 + 14 - 10 = 6$

~~Makes me scared...~~ ok

$$y = \begin{bmatrix} -7 \\ -2 \\ 6 \end{bmatrix}$$

Next solve $Ux=y$ by substitution...

$$U = \begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix} \qquad y = \begin{bmatrix} -7 \\ -2 \\ 6 \end{bmatrix}$$

Solve by substitution: (back substitution)

$$\begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7 \\ -2 \\ 6 \end{bmatrix}$$

$$\left.\begin{aligned} 3x_1 - 7x_2 - 2x_3 &= -7 \\ -2x_2 - x_3 &= -2 \\ -x_3 &= 6 \end{aligned}\right\}$$

$x_3 = -6$

$x_2 = \dfrac{x_3 - 2}{-2} = \dfrac{-6-2}{-2} = 4$

$x_1 = \dfrac{7x_2 + 2x_3 - 7}{3} = \dfrac{7 \cdot 4 + 2(-6) - 7}{3}$

Answer

$$x_1 = \frac{28-12-7}{3} = \frac{9}{3} = 3$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -6 \end{bmatrix}$$

Examples of finding the $A = \underset{3\times4}{} \underset{3\times3}{L}\, \underset{3\times4}{U}$ factorization:

the only size that fits

25#45

$$A = \begin{bmatrix} 2 & -4 & 4 & -2 \\ \textcircled{6} & -9 & 7 & -3 \\ \textcircled{-1} & -4 & 8 & 0 \end{bmatrix} \in \mathbb{R}^{3\times4}$$

$r_2 \leftarrow r_2 - 3r_1$

2,1 slot 3

$r_3 \leftarrow r_3 + \frac{1}{2} r_1$

row. col.

$$\begin{bmatrix} 2 & -4 & 4 & -2 \\ 0 & 3 & -5 & 3 \\ 0 & \textcircled{-6} & 10 & -1 \end{bmatrix}$$

$r_3 \leftarrow r_3 + 2r_2$

L is the matrix that undoes these elimination steps

since row operations are invertible then L must be square.

$$\begin{bmatrix} 2 & -4 & 4 & -2 \\ 0 & 3 & -5 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix} = U \in \mathbb{R}^{3\times4}$$

echelon form

what is L?

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1/2 & -2 & 1 \end{bmatrix}$$

How to solve a problem with this factorization

$LUx = b$

$$\begin{cases} Ly = b \\ Ux = y \end{cases}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1/2 & -2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$y_1 = 1$

$3y_1 + y_2 = 2 \qquad y_2 = 2-3 = -1$

$-\frac{1}{2}y_1 - 2y_2 + y_3 = 3$

$y_3 = 3 + \frac{1}{2}y_1 + 2y_2$

$= 3 + \frac{1}{2} - 2 = 2\frac{1}{2} = \frac{3}{2}$

$$y = \begin{bmatrix} 1 \\ -1 \\ 5/2 \end{bmatrix}$$

$$\begin{array}{cccc} P & P & F & P \\ \left[\begin{matrix} \textcircled{2} & -4 & 4 & -2 \\ 0 & \textcircled{3} & -5 & 3 \\ 0 & 0 & 0 & \textcircled{5} \end{matrix}\right. & & & \left.\right] \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3/2 \end{bmatrix}$$

Solve this by substitution...

or make augmented matrix

$$\begin{bmatrix} 2 & -4 & 4 & -2 & 1 \\ 0 & 3 & -5 & 3 & -1 \\ 0 & 0 & 0 & 5 & 3/2 \end{bmatrix}$$

make reduced echelon form to solve for x

$2x_1 - 4x_2 + 4x_3 - 2x_4 = 1$

$3x_2 - 5\,\textcircled{x_3} + 3x_4 = -1$

free, don't solve for x_3

$5x_4 = 3/2$

$x_4 = 3/10$

$$x_2 = \frac{-1 + 5x_3 - 3x_4}{3} = \frac{-1.9 + 5x_3}{3}$$

$$x_1 = \frac{1+4x_2-4x_3+2x_4}{2} =$$