Math 330: Quiz 1 Version D

This is a closed-book closed-notes no-calculator-allowed in-class exam. Efforts have been made to keep the arithmetic simple. If it turns out to be complicated, that's either because I made a mistake or you did. In either case, do the best you can and check your work where possible. While getting the right answer is nice, this is not an arithmetic test. It's more important to clearly explain what you did and what you know.

1. Indicate in writing that you have understood the requirement to work independently by writing "I have worked independently on this quiz" followed by your signature as the answer to this question.

2. Suppose $u, v \in \mathbf{R}^3$ and $A \in \mathbf{R}^{2\times 3}$ are given by

$$u = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \qquad v = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} \qquad \text{and} \qquad A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 3 \end{bmatrix}.$$

(i) Find $2u - v$.

(ii) Find Au.

3. Answer the following true false questions:

(i) An inconsistent system has more than one solution.

(A) True

(B) False

(ii) Whenever a system has free variables, the solution set contains a unique solution.

(A) True

(B) False

(iii) When two linear transformations are performed one after another, the combined effect may not always be a linear transformation.

(A) True

(B) False

(iv) Every elementary row operation is reversible.

(A) True

(B) False

4. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

5. Let A be a 3×2 matrix. Explain why the equation $Ax = b$ cannot be consistent for all b in $\mathbf{R}^3$. Generalize your argument to the case of an arbitrary A with more rows than columns.

6. Write down the augmented matrix $\left[\, A \,|\, b \,\right]$ corresponding to the system of linear equations given by

$$\begin{cases} -x_1 - 3x_2 + 5x_3 - 3x_4 = 7 \\ x_1 + 5x_3 + 9x_4 = -2 \\ 3x_1 + 4x_2 - 2x_3 - x_4 = 1 \end{cases}$$

but *do not* solve these equations.

7. Suppose $A \in \mathbf{R}^{2\times 3}$ is given by

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix}.$$

How many free variables does the equation $Ax = 0$ have? Find all solutions to the equation $Ax = 0$.

8. The LU factorization of a matrix A is given by

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 5/3 & -2 & 1 \end{bmatrix} \qquad \text{and} \qquad U = \begin{bmatrix} -3 & 3 & 6 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{bmatrix}.$$

Explain how to use this factorization to solve the equation $Ax = b$ and then find the value of x corresponding to $b = (6, 7, -1)$.