Math 330: Exam 2 Version B

This is a closed-book closed-notes no-calculator-allowed in-class exam. Efforts have been made to keep the arithmetic simple. If it turns out to be complicated, that's either because I made a mistake or you did. In either case, do the best you can and check your work where possible. While getting the right answer is nice, this is not an arithmetic test. It's more important to clearly explain what you did and what you know.

1. Indicate in writing that you have understood the requirement to work independently by writing "I have worked independently on this quiz" followed by your signature as the answer to this question.

2. Consider the matrix A with reduced row eschelon form R where

$$A = \begin{bmatrix} \frac{4}{3} & -1 & -3 & -3 & \frac{5}{2} \\ \frac{8}{3} & -2 & -6 & -\frac{7}{2} & \frac{17}{3} \\ \frac{16}{9} & -\frac{4}{3} & -4 & \frac{7}{2} & \frac{10}{3} \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & -\frac{3}{4} & -\frac{9}{4} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   (i) Find a basis for Col(A).

   (ii) Find a basis for Nul(A).

3. Answer the following true false questions:

   (i) $\det(A^{-1}) = (-1)\det(A)$.

     (A) True

     (B) False

  (ii) Cramer's rule can only be used for invertible matrices.

     (A) True

     (B) False

  (iii) If the columns of an $m \times n$ matrix A are orthonormal, then the linear mapping $x \to Ax$ preserves lengths.

     (A) True

     (B) False

  (iv) If $A = QR$ where Q has orthonormal columns, then $R = Q^T A$.

     (A) True

     (B) False

4. Let U be a square matrix such that $U^T U = I$. Show that $\det(U) = \pm 1$.

5. What is the rank of a 4×5 matrix whose null space is two dimensional?

6. Find $\det(A)$, $\det(B)$ and $\det(AB)$ where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & 3 \\ 0 & 0 & 5 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

7. Suppose $A \in \mathbf{R}^{2 \times 2}$ is given by

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}.$$

Use the Gram-Schmidt algorithm to factor $A = QR$ where Q is a matrix with orthonormal columns and R is upper triangular.

8. The QR factorization of a matrix A is given by

$$
Q = \begin{bmatrix} \dfrac{1}{3} & \dfrac{2}{3\sqrt{5}} \\[2ex] -\dfrac{2}{3} & \dfrac{\sqrt{5}}{3} \\[2ex] \dfrac{2}{3} & \dfrac{4}{3\sqrt{5}} \end{bmatrix}
\quad \text{and} \quad
R = \begin{bmatrix} 3 & \dfrac{1}{3} \\[2ex] 0 & \dfrac{4\sqrt{5}}{3} \end{bmatrix}.
$$

Explain how to use this factorization to minimize $\|Ax - b\|$ and then find the minimizing value of x corresponding to $b = (1, 0, 1)$.