section 1004
Jan 19, '22

Chapter 1.1

A **linear equation** in the variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$\boxed{a_1x_1 + a_2x_2 + \cdots + a_nx_n = b} \quad \leftarrow \text{General form} \qquad (1)$$

where b and the **coefficients** $a_1, \ldots, a_n$ are real or complex numbers, usually known in advance. The subscript n may be any positive integer. In textbook examples and

Specific

$$\left\{ \begin{array}{l} \boxed{\begin{array}{l} 2x + 3y \\ 7x - 5y \end{array}} \begin{array}{l} = 4 \\ = 6 \end{array} \end{array} \right.$$

↑ Identify this as a linear function

$f(x,y) = (2x + 3y,\ 7x - 5y)$

Solve this...

$f(x,y) = (4,6)$

Notation

$$f(x,y) = \begin{bmatrix} 2x + 3y \\ 7x - 5y \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 7 & -5 \end{bmatrix}$$ ← distinguish the exact function by the numbers...

System of linear equations

The left side is a linear function

$$\begin{cases} 2x - 3y = 7 \\ 5x + 4y = 3 \end{cases}$$

Eliminate x

Take first eq. and mult by $\frac{5}{2}$

$$5x + 4y = 3$$
$$5x - \frac{15}{2}y = \frac{35}{2}$$

$$\frac{23}{2}y = \frac{-29}{2}$$

$$y = \frac{-29}{23}$$

substitute back into find x...

$$f(x,y) = (2x - 3y, 5x + 4y)$$

The equation becomes

$$f(x,y) = (7,3)$$

More notation...

$$f(x,y) = \begin{bmatrix} 2x - 3y \\ 5x + 4y \end{bmatrix} \qquad A = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$$

System of eq:

$$\begin{bmatrix} 2x-3y \\ 5x+4y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

linear function $f(x,y)$

$$g(x,y) = \begin{bmatrix} 1x - 2y \\ 5x + 3y \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$$

$$(f\circ g)(x,y) = f(g(x,y)) = f(1x-2y,\ 5x+3y)$$

$$= \begin{bmatrix} 2(1x-2y)-3(5x+3y) \\ 5(1x-2y)+4(5x+3y) \end{bmatrix} = \begin{bmatrix} -13x - 13y \\ 25x + 2y \end{bmatrix}$$

$$C = \begin{bmatrix} -13 & -13 \\ 25 & 2 \end{bmatrix}$$

By definition

$C = AB$.