

# Contents

Preface **viii**

A Note to Students **xv**

## Chapter 1 Linear Equations in Linear Algebra 1

INTRODUCTORY EXAMPLE: Linear Models in Economics and Engineering 1

1.1	Systems of Linear Equations	2
1.2	Row Reduction and Echelon Forms	12
1.3	Vector Equations	24
1.4	The Matrix Equation $Ax = b$	35
1.5	Solution Sets of Linear Systems	43
1.6	Applications of Linear Systems	50
1.7	Linear Independence	56
1.8	Introduction to Linear Transformations	63
1.9	The Matrix of a Linear Transformation	71
<del>1.10</del>	Linear Models in Business, Science, and Engineering	81
	Supplementary Exercises	89

matrix  
 $A = LU$   
 Gaussian Elimination  
 two other matrices

factorization

The matrices  $L$  and  $U$  are simpler than  $A$ .

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 5 & 0 \\ -1 & 7 & 13 \end{bmatrix}$$

lower triangular

Matrix is a bunch of numbers between same brackets

$$\begin{bmatrix} 1 & 2 & 6 \\ -2 & 3 & 0 \\ 5 & 1 & -2 \end{bmatrix}$$

## Chapter 2 Matrix Algebra 93

INTRODUCTORY EXAMPLE: Computer Models in Aircraft Design 93

2.1	Matrix Operations	94
2.2	The Inverse of a Matrix	104
2.3	Characterizations of Invertible Matrices	113
2.4	Partitioned Matrices	119
<u>2.5</u>	<u>Matrix Factorizations</u>	<u>125</u>
<del>2.6</del>	The Leontief Input-Output Model	134
<del>2.7</del>	Applications to Computer Graphics	140
<u>2.8</u>	<u>Subspaces of <math>\mathbb{R}^n</math></u>	<u>148</u>
<u>2.9</u>	<u>Dimension and Rank</u>	<u>155</u>
	Supplementary Exercises	162

interpretation is that a matrix represent a linear function...

Algebra means we have rules to add and multiply matrices... function composition...

focus  
 vector spaces → 2.8  
 2.9

## Chapter 3 Determinants 165

INTRODUCTORY EXAMPLE: Random Paths and Distortion 165

3.1	Introduction to Determinants	166
3.2	Properties of Determinants	171
3.3	Cramer's Rule, Volume, and Linear Transformations	179
	Supplementary Exercises	188

will be used in chapter 5

## Chapter 4 Vector Spaces 191

### INTRODUCTORY EXAMPLE: Space Flight and Control Systems 191

- 4.1 Vector Spaces and Subspaces 192
- 4.2 Null Spaces, Column Spaces, and Linear Transformations 200
- 4.3 Linearly Independent Sets; Bases 210
- 4.4 Coordinate Systems 218
- 4.5 The Dimension of a Vector Space 227
- 4.6 Rank 232
- 4.7 Change of Basis 241
- 4.8 Applications to Difference Equations 246
- 4.9 Applications to Markov Chains 255
- Supplementary Exercises 264

## Chapter 5 Eigenvalues and Eigenvectors 267

### INTRODUCTORY EXAMPLE: Dynamical Systems and Spotted Owls 267

- 5.1 Eigenvectors and Eigenvalues 268
- 5.2 The Characteristic Equation 276
- 5.3 Diagonalization 283
- 5.4 Eigenvectors and Linear Transformations 290
- 5.5 Complex Eigenvalues 297
- 5.6 Discrete Dynamical Systems 303
- 5.7 Applications to Differential Equations 313
- 5.8 Iterative Estimates for Eigenvalues 321
- Supplementary Exercises 328

Important in diff. eq. and many other places

use determinants

$$A = SDS^{-1}$$

inverse functions of the function corresponding to  $S$ .  
diagonal matrix

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -7 \end{bmatrix}$$

## Chapter 6 Orthogonality and Least Squares 331

### INTRODUCTORY EXAMPLE: The North American Datum and GPS Navigation 331

- 6.1 Inner Product, Length, and Orthogonality 332
- 6.2 Orthogonal Sets 340
- 6.3 Orthogonal Projections 349
- 6.4 The Gram-Schmidt Process 356
- 6.5 Least-Squares Problems 362
- 6.6 Applications to Linear Models 370
- 6.7 Inner Product Spaces 378
- 6.8 Applications of Inner Product Spaces 385
- Supplementary Exercises 392

$$f(x, y, z) = (2x, 4y, -7z)$$

$$AA = SDS^{-1}SDS^{-1}$$

$$AA = SDD S^{-1}$$

$$DD = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 49 & 0 \end{bmatrix}$$

optimization problems

Gram-Schmidt.

$$A = QR$$

orthogonal matrix      upper triangular matrix

$Q^T = Q^{-1}$   
Switch rows with columns.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$f(x, y, z) = (2x, 4y, -7z)$$

DD means compose  $f \circ f$

$$\begin{aligned} \underbrace{(f \circ f)(x, y, z)} &= f(f(x, y, z)) = f(2x, 4y, -7z) = (2(2x), 4(4y), -7(-7z)) = \\ &= (4x, 16y, 49z) \end{aligned}$$

$$DD = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 49 \end{bmatrix}$$

## Chapter 7 Symmetric Matrices and Quadratic Forms 395

<b>INTRODUCTORY EXAMPLE: Multichannel Image Processing</b>		<b>395</b>
7.1	Diagonalization of Symmetric Matrices	<b>397</b>
7.2	Quadratic Forms	<b>403</b>
7.3	Constrained Optimization	<b>410</b>
7.4	The Singular Value Decomposition	<b>416</b>
7.5	Applications to Image Processing and Statistics	<b>426</b>
	Supplementary Exercises	<b>434</b>

## Chapter 8 The Geometry of Vector Spaces 437

<b>INTRODUCTORY EXAMPLE: The Platonic Solids</b>		<b>437</b>
8.1	Affine Combinations	<b>438</b>
8.2	Affine Independence	<b>446</b>
8.3	Convex Combinations	<b>456</b>
8.4	Hyperplanes	<b>463</b>
8.5	Polytopes	<b>471</b>
8.6	Curves and Surfaces	<b>483</b>

## Chapter 9 Optimization (Online)

<b>INTRODUCTORY EXAMPLE: The Berlin Airlift</b>	
9.1	Matrix Games
9.2	Linear Programming—Geometric Method
9.3	Linear Programming—Simplex Method
9.4	Duality

## Chapter 10 Finite-State Markov Chains (Online)

<b>INTRODUCTORY EXAMPLE: Googling Markov Chains</b>	
10.1	Introduction and Examples
10.2	The Steady-State Vector and Google's PageRank
10.3	Communication Classes
10.4	Classification of States and Periodicity
10.5	The Fundamental Matrix
10.6	Markov Chains and Baseball Statistics

## Appendixes

A	Uniqueness of the Reduced Echelon Form	<b>A1</b>
B	Complex Numbers	<b>A2</b>

*Glossary* **A7**

*Answers to Odd-Numbered Exercises* **A17**

*Index* **I1**

*Photo Credits* **P1**

$A = U \Sigma V^T$

diagonal

orthogonal

orthogonal

end of course