The eigenvalue eigenvector program transforms matrix vector multiplication into scalar vector multiplication...that's much simpler...

If x is an eigenvector for AEIR<sup>nxn</sup> and A: is the corresponding eigenvalue, to scalar...

This is n equations with not 1

 $Ax = \lambda x$ 

you are solving for both 2 and a.

I dea given A then solve for x and 2

 $Ax - \lambda x = 0$ 

 $(A-\lambda I)x=0$ 

exclude the solution x=0 because it's not useful.

B=A-xt

Solving Bx=0 (homogeneous equation) the 2015 which satisfy this Nul(B) = 2 = 8x=03.

B must have free voriable for there to be a non-zero solution x to Bz=0.

For B to have free variables means B is not inertible

## If B is not invertible than det B=0. (if det B #0 then Cramer's rule) (would imply B was invertible ...)

B=A->t

Need det  $(A - \lambda I) = 0$ 

Called characteristic equation.

Characteristic polynomial X(2)=det(A-2I)

Thus \$(1)=0 is the characteristic equation.

rample:

Toying to solve 
$$\chi(\lambda) = 0$$

13.  $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \lambda = 1,2,3$ 

i.e. verify that  $\chi(1) = 0$ 
 $\chi(2) = 0$  and  $\chi(3) = 0$ 

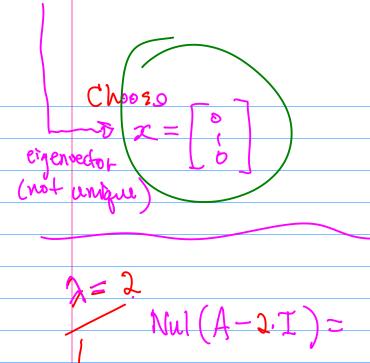
$$A(\lambda) = \operatorname{det}(A-\lambda I) = \operatorname{det}\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \sim \lambda \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= du + \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$$

Note setting 2=1 yields

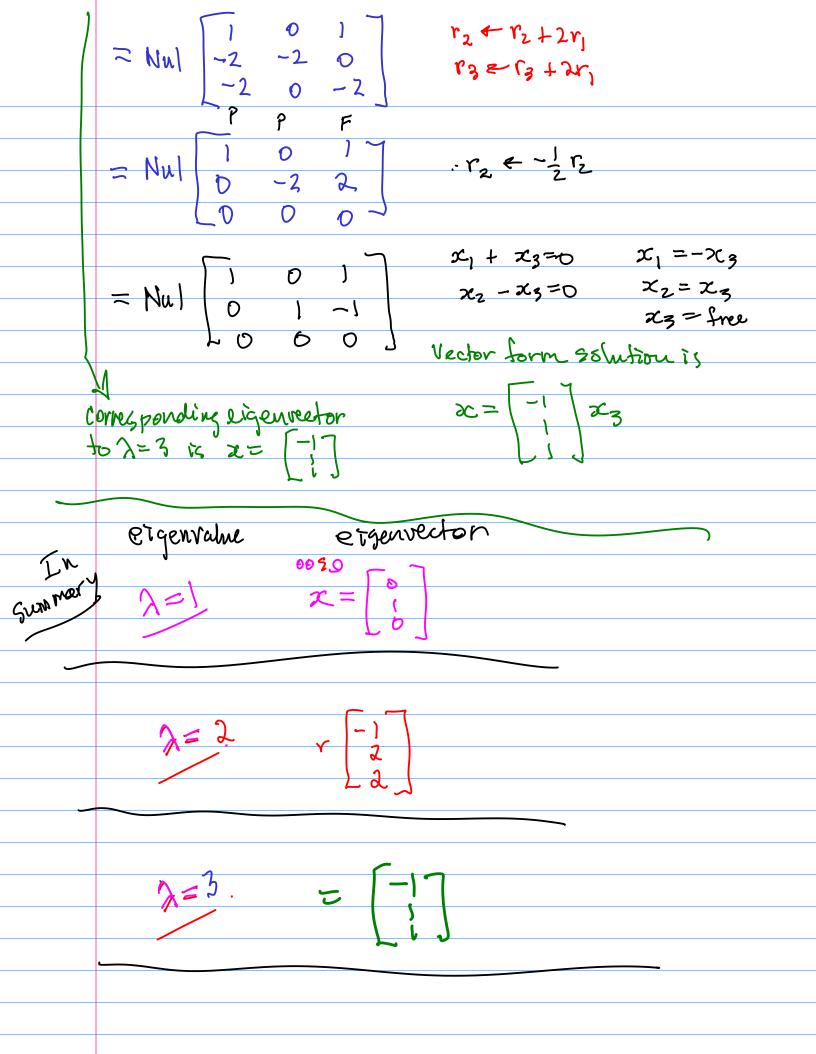
difficult to use Gaussian elimination to find the determinant, because the varioble of is unknown 5- it's difficult to know what terms are

$$\begin{array}{l} \chi(\lambda) = dut \begin{cases} 4-\lambda & 0 \\ -2 & 1-\lambda \end{cases} \\ = -0 \ det \ B_{12} + (1-\lambda) \ det \ B_{22} - 0 \ det \ B_{32} \\ = (1-\lambda) \ det \begin{cases} 4-\lambda & 1 \\ -2 & 1-\lambda \end{cases} = (1-\lambda) \left( (4-\lambda)(1-\lambda) + 2 \right) \\ = (1-\lambda) \left( 4-5\lambda + \lambda^2 + 2 \right) = (1-\lambda) \left( \lambda^2 - 5\lambda + 6 \right) \\ = (1-\lambda) \left( \lambda - 3 \right) (\lambda - 2) = 0 \quad \text{yield} \quad \lambda = 1, 2 \text{ or } 3. \\ \text{The this algorithm to factor the matrix } A \\ \text{First let is solve for the active to the matrix } A \\ \text{First let is solve for the active to 1 } \\ \text{First let is solve for the active to 2 } \\ \text{Mul} \left( A - 1 \cdot T \right) = \text{Nul} \begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} = \begin{cases} x_1 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{cases} \\ \text{Nul} \left( x_2 & 0 & 0 \\ -2 & 0 & 0 \end{cases} = \begin{cases} 3x_1 + x_3 = 0 & x_1 = 0 \\ -2x_1 & 0 & 0 \\ -2x_1 & 0 & x_2 = 0 \end{cases}$$



any of these 2's are eigenvectors for >= 1

$$3 = 3$$
 $Nul(A-3I) = Nul \begin{bmatrix} 4-3 & 0 & 1 \\ -2 & 1-3 & 0 \\ -2 & 0 & 1-3 \end{bmatrix}$ 



Use this to factor the matrix A

$$= \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \approx \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ 0 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 3 \\ 0 & 3 & 5 \end{bmatrix} \begin{bmatrix} -1 & 3 & -1 \\ 1 & 2 & 3 \\ 0 & 3 & 5 \end{bmatrix}$$

$$5 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ 0 & 3 & 1 \end{bmatrix} \qquad \begin{array}{c} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{array}$$

) thus A=5D5-1

\* factorization