Section 1004
Jan 26'22

original system of linear equations

$$\begin{cases} 2x - 3y = 7 \\ 5x + 4y = 3 \end{cases}$$

- LU factorization to convert this system into two simpler system.

$$\underbrace{\begin{cases} a = 7 \\ \frac{5}{2}a + b = 3 \end{cases}}^{L} \qquad \underbrace{\begin{cases} 2x - 3y = a = 7 \\ \frac{23}{2}y = b = \frac{-29}{2} \end{cases}}^{U}$$

Solve by substitution...

$b = 3 - \frac{5}{2}7 = \frac{6-35}{2} = \frac{-29}{2}$

$(a,b) = (7, \frac{-29}{2})$

by substitution

$y = \frac{-29}{23}$

$x = \frac{7 + 3\cdot(\frac{-29}{23})}{2} = ?$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$x + 2y + 3z = 1$
$4x + 5y + 6z = 1$
$7x + 8y + 9z = 1$

9
-21
-12

elimination steps to find U (row eschelon form of A).

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow[r_3 \leftarrow r_3 - 7r_1]{r_2 \leftarrow r_2 - 4r_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \xrightarrow{r_3 \leftarrow r_3 - 2r_2} \begin{array}{c} \begin{array}{ccc} P & P & F \end{array} \\ \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}}_{U} \end{array}$$

what's L? It's the matrix that performs the row operations

- $r_3 \leftarrow r_3 + 2r_2$
- $r_3 \leftarrow r_3 + 7r_1$
- $r_2 \leftarrow r_2 + 4r_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad r_3 \leftarrow r_3 + 2r_2 \quad \begin{bmatrix} \boxed{1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \boxed{2} & 1 \end{bmatrix} \quad r_3 \leftarrow r_3 + 7r_1 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{7} & 2 & 1 \end{bmatrix}$$

3rd row, 2nd col; 3rd row, 1st col.; times 7

$$r_2 \leftarrow r_2 + 4r_1 \quad \begin{bmatrix} 1 & 0 & 0 \\ \boxed{4} & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} = L \qquad A = LU$$

2nd row, 1st col.

undoes the row op. that created U from A.

I have this matrix factorization:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \boxed{4} & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \overset{U}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}}$$

from L

$$\begin{cases} a = 1 \\ 4a + b = 1 \\ 7a + 2b + c = 1 \end{cases}$$

and

from U

$$\begin{cases} x + 2y + 3z = a = 1 \\ -3y - 6z = b = -3 \\ 0z = c \end{cases}$$

$b = 1 - 4\cdot 1 = -3$

$c = 1 - 7\cdot 1 - 2(-3) = 0$ $\boxed{\text{lucky}}$

z is called a free variable. because it can be anything..

substitution

$$y = \frac{-3 + 6z}{-3} = 1 - 2z$$

$$x = 1 - 2(1 - 2z) - 3z$$

$$x = -1 + z$$

ANSWER

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 + z \\ 1 - 2z \\ z \end{bmatrix}$$ for any z I like

many different answers, one for each value of z. (A one dimensional subset of answers).

Try a different right hand side

from L

$$\begin{cases} a = 1 \\ 4a + b = 1 \\ 7a + 2b + c = 2 \end{cases}$$

$b = 1 - 4 \cdot 1 = -3$

$c = 2 - 7 \cdot 1 - 2(-3) = 1$

and

from U

$$\begin{cases} x + 2y + 3z = a = 1 \\ -3y - 6z = b = -3 \\ 0 = c = 1 \end{cases}$$

unlucky

inconsistent

z is called a free variable because it can be anythi

In this case there are no solutions to the problem...

1.2 Augmented matrices

coefficient matrix

right hand side of the system

$$\textbf{12.} \quad \left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 0 & 7 & -4 & 2 \end{array}\right]$$

System of equations

awkward idea, but explains why everything work as before...

$$\begin{cases} 1x_1 - 7x_2 + 0x_3 + 6x_4 + 5(x_5) = 0 \\ 0x_1 + 0x_2 + 1x_3 - 2x_4 + -3(x_5) = 0 \\ -1x_1 + 7x_2 - 4x_3 + 2x_4 + 7(x_5) = 0 \\ x_5 = -1 \end{cases}$$

System of equations

$$1x_1 - 7x_2 + 0x_3 + 6x_4 = 5$$
$$0x_1 + 0x_2 + 1x_3 - 2x_4 = -3$$
$$-1x_1 + 7x_2 - 4x_3 + 2x_4 = 7$$

$$\textbf{12.} \quad \left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{array}\right]$$

$r_3 \leftarrow r_3 + r_1$

$$\left[\begin{array}{cccc|c} \boxed{1} & -7 & 0 & 6 & 5 \\ 0 & 0 & \boxed{1} & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{array}\right]$$