Section 1005
Jan 27, '22

From last time ...

$$A=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}$$

$r_2 \leftarrow r_2 - 4r_1$ ✓

$\boxed{r_3 \leftarrow r_3 - 7r_1}$ ✓

$$\begin{bmatrix}1&2&3\\0&-3&-6\\0&-6&-12\end{bmatrix}$$

$\boxed{r_3 \leftarrow r_3 - 2r_2}$

So 2 goes in the (3,2) slot of L

$$\begin{array}{cccc} & P & P & F \\ U= & \boxed{1} & 2 & 3 \\ & 0 & \boxed{-3} & -6 \\ & 0 & 0 & 0 \end{array}$$

To get back to A from U undo each of these row operations one at a time starting with the last and ending with the first.

$r_3 \leftarrow r_3 + 2r_2$

whatever this function is, say it's g then $g(id(x,y,z)) = g(x,y,z)$

write $id(x,y,z)$ as a matrix and plug that in here

out comes the matrix for g here.

$r_3 \leftarrow r_3 + 7r_1$

$r_2 \leftarrow r_2 + 4r_1$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \xrightarrow{r_3 \leftarrow r_3 + \boxed{2} r_2} \begin{bmatrix}1&0&0\\0&1&0\\0&2&1\end{bmatrix} \xrightarrow{r_3 \leftarrow r_3 + \boxed{7} r_1}$$

row col (3,2) slot

row col of where the 7 goes

$$\begin{array}{cccc} & \boxed{1} & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ \boxed{3} & 7 & 2 & 1 \end{array} \xrightarrow{r_2 \leftarrow r_2 + 4r_1} \begin{bmatrix}1&0&0\\4&1&0\\7&2&1\end{bmatrix} = L$$

Therefore $A = LU$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

Solving the original system is the same a solving

$$\begin{cases} 1a = 1 \\ 4a + b = 2 \\ 7a + 2b + c = 3 \end{cases}$$

pivot variables

$$\begin{cases} \boxed{1x} + 2y + 3z = a = 1 \\ \boxed{-3y} - 6z = b = -2 \\ 0z = c = 0 \end{cases}$$

on other side

Solve by substitution..

$b = 2 - 4a = 2 - 4 \cdot 1 = -2$

$c = 3 - 7a - 2b = 3 - 7 \cdot 1 - 2(-2)$

$= 3 - 7 + 4 = 0$

this compatibility condition is ok, by chance...

z can be anything... It's a free variable...

Solve by substitution

$y = \dfrac{-2 + 6z}{-3} = \dfrac{2}{3} - 2z$

$x = 1 - 2y - 3z$

$= 1 - 2\left(\dfrac{-2 + 6z}{-3}\right) - 3z$

$x = -\dfrac{1}{3} + z$

Solution:

$$\begin{cases} x = -\frac{1}{3} + z \\ y = \frac{2}{3} - 2z \\ z = z \end{cases}$$

for any value of z you like.

There is an infinite number of solutions to the problem...

$$\begin{aligned} x + 2y + 3z &= 1 \\ 4x + 5y + 6z &= 2 \\ 7x + 8y + 9z &= 3 \end{aligned}$$

$$\begin{cases} 1a = 1 \\ 4a + b = 2 \\ 7a + 2b + c = 4 \end{cases}$$

1 o-variable

$$\begin{cases} \boxed{1x} + 2y + 3z = a = 1 \\ \boxed{-3y} - 6z = b = -2 \\ 0z = c = 1 \end{cases}$$

on other side

inconsistent

No solution!

Solve by substitution..

$b = 2 - 4a = 2 - 4 \cdot 1 = -2$ ✓

$c = 4 - 7a - 2b = 4 - 7 \cdot 1 - 2(-2)$

$= 4 - 7 + 4 = 1$

this compatibility condition is not ok

Section 1.2 Augmented matrices

7. $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right]$

$$\begin{cases} 1x_1 + 3x_2 + 4x_3 = 7 \\ 3x_1 + 9x_2 + 7x_3 = 6 \end{cases}$$

Matrix, that is the linear function

right hand side of the system

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

$r_2 \leftarrow r_2 - 3r_1$

the pivots

Free vbl

row eschelon form.

$$\begin{bmatrix} \boxed{1} & 3 & 4 & 7 \\ 0 & 0 & \boxed{-5} & -15 \end{bmatrix} = U$$

one could rewrite as

$\boxed{1x_1} + 3x_2 + 4x_3 = 7$

$\boxed{-5x_3} = -15$

$x_3 = 3$

don't have to divide by -5

$x_1 = 7 - 3x_2 - 4x_3$

$= 7 - 3x_2 - 12 = -5 - 3x_2$

free vbl. infinite number of soln's.

reduced row eschelon form

$r_1 \leftarrow r_1 + \frac{4}{5} r_2$

$$\begin{bmatrix} \boxed{1} & 3 & 0 & -5 \\ 0 & 0 & \boxed{-5} & -15 \end{bmatrix}$$

already did that in the reduced row esch. form..

Now rescale the pivots so they are 1.

$r_2 \leftarrow \frac{-1}{5} r_2$

There it is

$$\begin{bmatrix} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

use to solve the system

$x_1 + 3x_2 + 0x_3 = -5$

$x_3 = 3$

$x_1 = -5 - 3x_2$