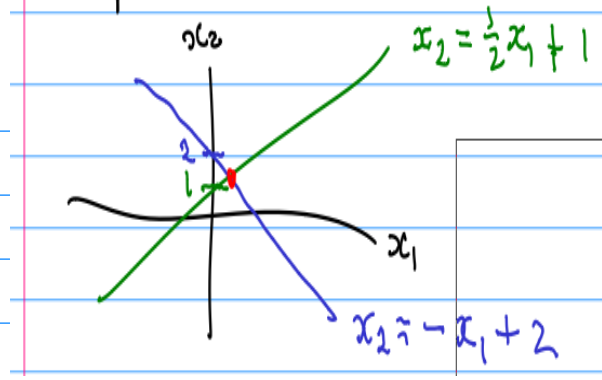
Example:



$$\begin{cases} \frac{1}{2}x_1 - x_2 = -1 \\ x_1 + x_2 = 2 \end{cases}$$

$$A = \begin{bmatrix} 1/2 & -1 \\ 1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \qquad Ax = b$$

Solve

$2\left(\frac{1}{2}x_1 - x_2\right) = (-1)(2)$ mult by 2

$x_1 + x_2 = 2$

scaling operation

$x_1 - 2x_2 = -2$

$x_1 + x_2 = 2$ subtract equation

$-3x_2 = -4$ elimination step

$x_2 = 4/3$

$x_1 - 2 \cdot 4/3 = -2$ substitute x_2 into first eq

$x_1 = \frac{8}{3} - 2 = \frac{2}{3}$ still call it substitution

Solution $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 4/3 \end{bmatrix}$,

$A = \begin{bmatrix} 1/2 & -1 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

ELEMENTARY ROW OPERATIONS

elimination..

1. (Replacement) Replace one row by the sum of itself and a multiple of another row.[1]
2. (Interchange) Interchange two rows.
3. (Scaling) Multiply all entries in a row by a nonzero constant.

Gaussian Elimination

$$\left[\begin{array}{c|c} A & b \end{array}\right]$$

augmented matrix

$$\begin{bmatrix} 1/2 & -1 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

scaling $r_1 \leftarrow 2r_1$

row 1

replaced by

$x_1 - 2x_2 = -2$

$x_1 + x_2 = 2$

$$\begin{bmatrix} 1 & -2 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

elimination

$r_2 \leftarrow r_2 - r_1$

$-3x_2 = -4$

$$\begin{array}{cc} x_1 & x_2 \\ \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -2 & -2 \\ 0 & 3 & 4 \end{array}\right]$$

LHS RHS

substitutions

$$\begin{cases} x_1 - 2x_2 = -2 \\ \quad\;\; 3x_2 = 4 \end{cases}$$

$x_2 = 4/3$

$x_1 = 2/3$

Note we solved for x_2 and then for x_1 ...

back substitution

because solve for the x's in reverse order.

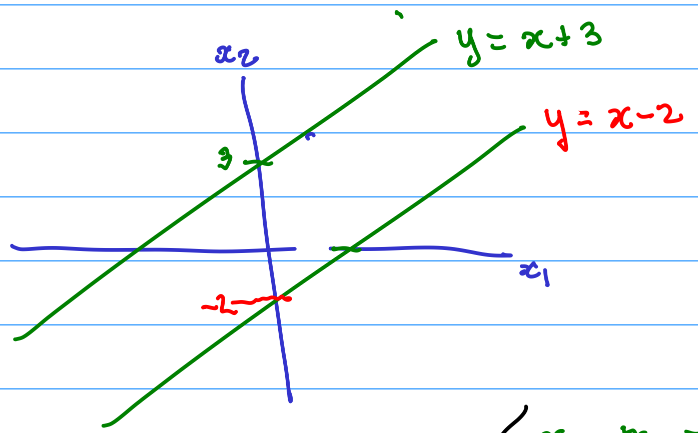
Things that can happen when solving systems of linear eqns.

A system of linear equations has

1. no solution, or
2. exactly one solution, or ☑ seen an example of this
3. infinitely many solutions.

well posed problem has exactly one solution...

Example with no solution



parallel lines don't intersect so there is no simultaneous solution...

System $\begin{cases} x_1 - x_2 = -3 \\ x_1 - x_2 = 2 \end{cases}$

$A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

augmented matrix

$$[A \mid b] = \begin{bmatrix} 1 & -1 & -3 \\ 1 & -1 & 2 \end{bmatrix}$$

do elimination steps

$r_2 \leftarrow r_2 - r_1$

$$\begin{array}{ccc} x_1 & x_2 & RHS \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -1 & -3 \\ 0 & 0 & 5 \end{array}\right]$$

reinterpret as a system

$$x_1 - x_2 = -3$$
$$0 = 5$$

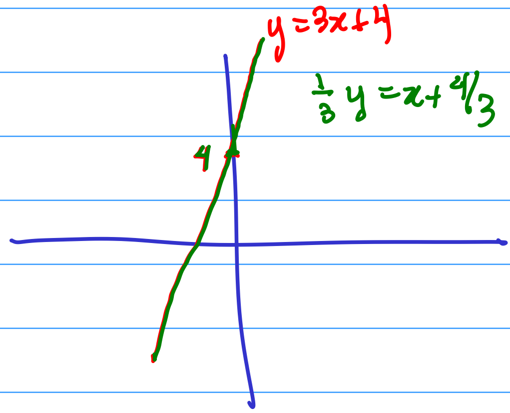
← contradiction...

conclusion: No Solution.

Infinitely many solutions

$$\begin{cases} y = 3x + 4 \\ \frac{1}{3}y = x + 4/3 \end{cases}$$

Same equation, just written differently.

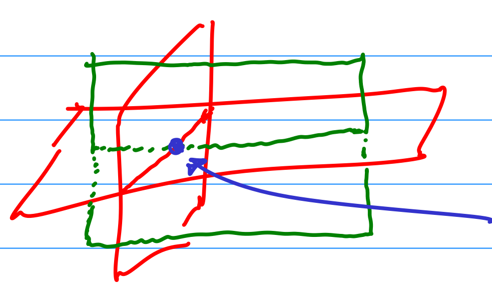


the lines intersect everywhere because the are the same.

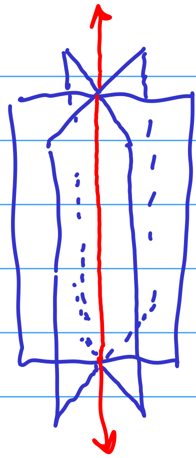
Seems easy in 2D, what about 3D?

3 equations in 3-space or 3 planes in 3 space

solution set for each equation is a



all three plane meet at one point...



3 plane... each one is different
and the meet along a line...
So infinite number of
simultaneous solutions...

ELEMENTARY ROW OPERATIONS = use for Gaussian Elimination and making LU factorization...

1. (Replacement) Replace one row by the sum of itself and a multiple of another row.[1] $r_i \leftarrow r_i + \alpha r_j$ $\quad i \neq j$
2. (Interchange) Interchange two rows. $r_i \leftrightarrow r_j$ $\quad i \neq j$
3. (Scaling) Multiply all entries in a row by a nonzero constant. $r_i \leftarrow \alpha r_i$ where $\alpha \neq 0$

ELEMENTARY ~~ROW~~ Column OPERATIONS = use for Gram-Schmidt orthogonalization and making QR factorization.

1. (Replacement) Replace one ~~row~~ column by the sum of itself and a multiple of another ~~row~~ column.[1]
2. (Interchange) Interchange two ~~rows~~ columns.
3. (Scaling) Multiply all entries in a ~~row~~ column by a nonzero constant.