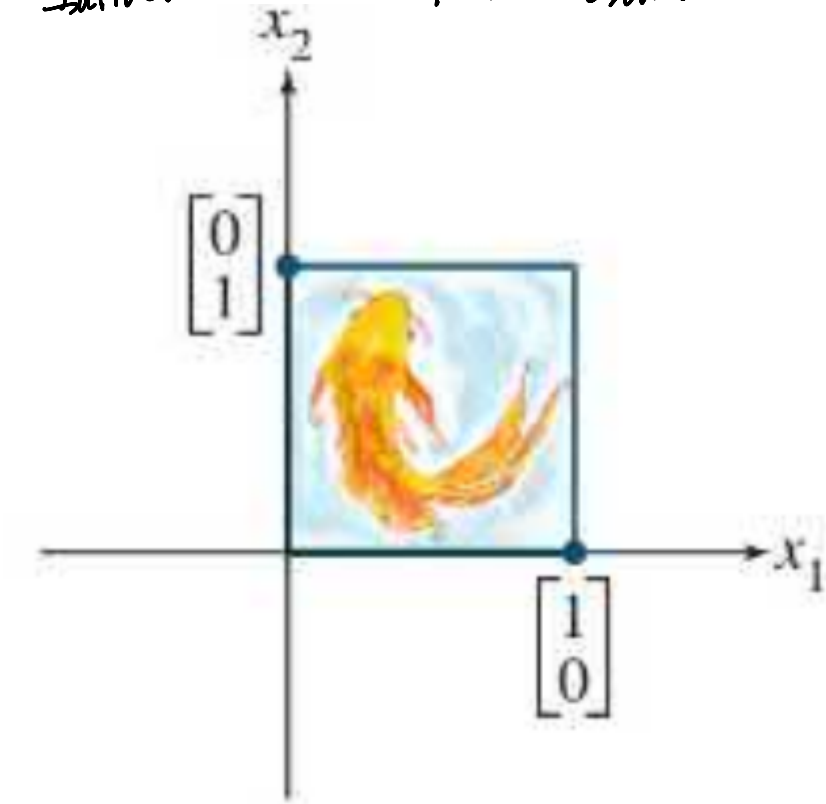
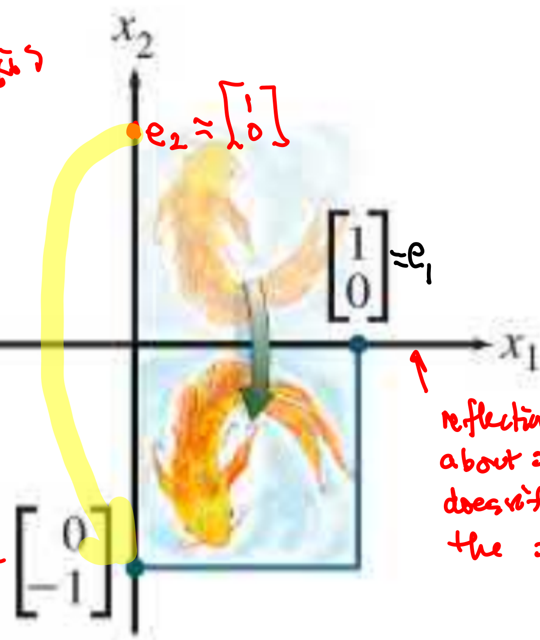
Initial condition of the fish...



IGURE 2

After applying a linear tranformation we get

Reflection about x_1-axis



$e_2 = \begin{bmatrix}1\\0\end{bmatrix}$

$= e_1$

$T(e_2) = \begin{bmatrix}0\\-1\end{bmatrix}$

reflecting about x_1-axis doesn't change the x_1-axis...

What matrix corresponds to this linear function?

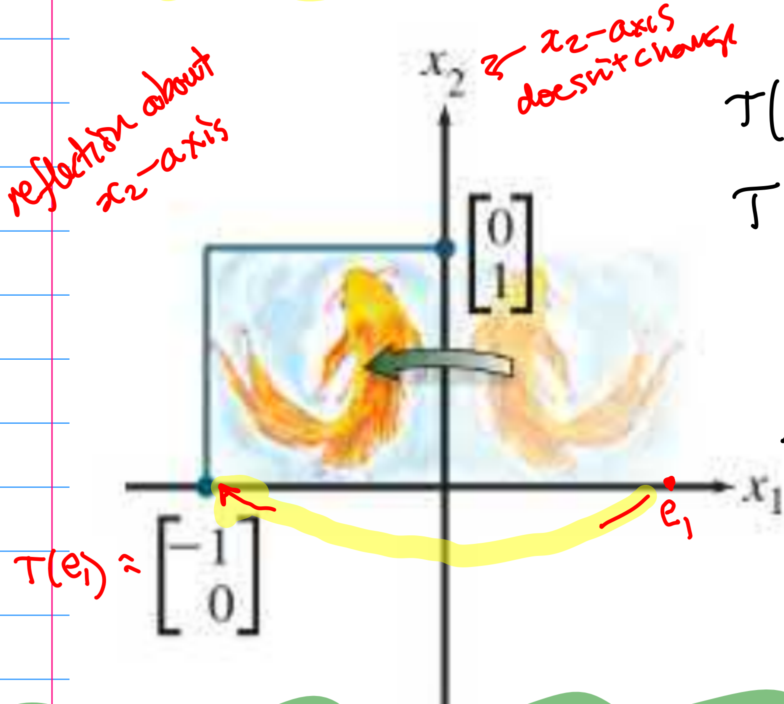
$$A = \left[\, T(e_1) \;\middle|\; T(e_2) \,\right]$$

$$T(e_1) = e_1 = \begin{bmatrix}1\\0\end{bmatrix}$$

$$T(e_2) = \begin{bmatrix}0\\-1\end{bmatrix}$$

Therefore $A = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$

Note given an linear function T there is exactly one A the represents that function.

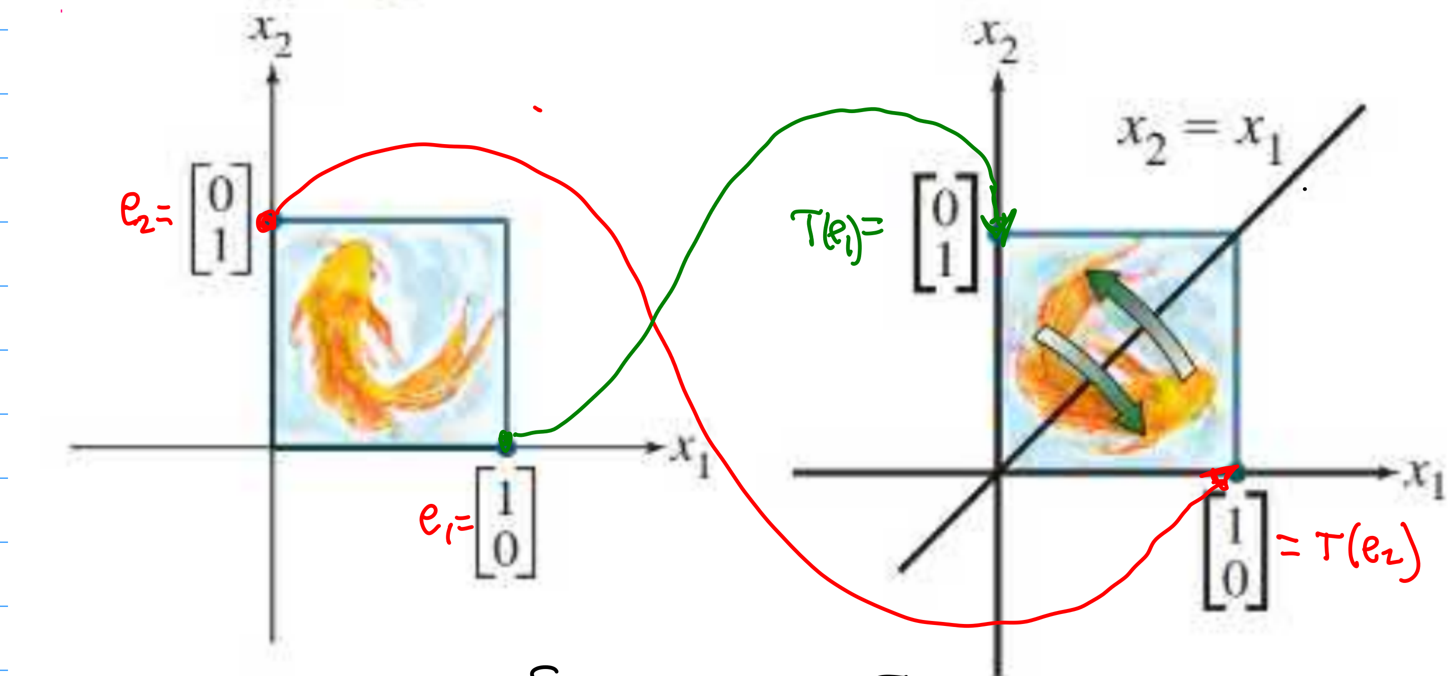


$T(e_2) = e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$A = \left[T(e_1) \middle| T(e_2) \right] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

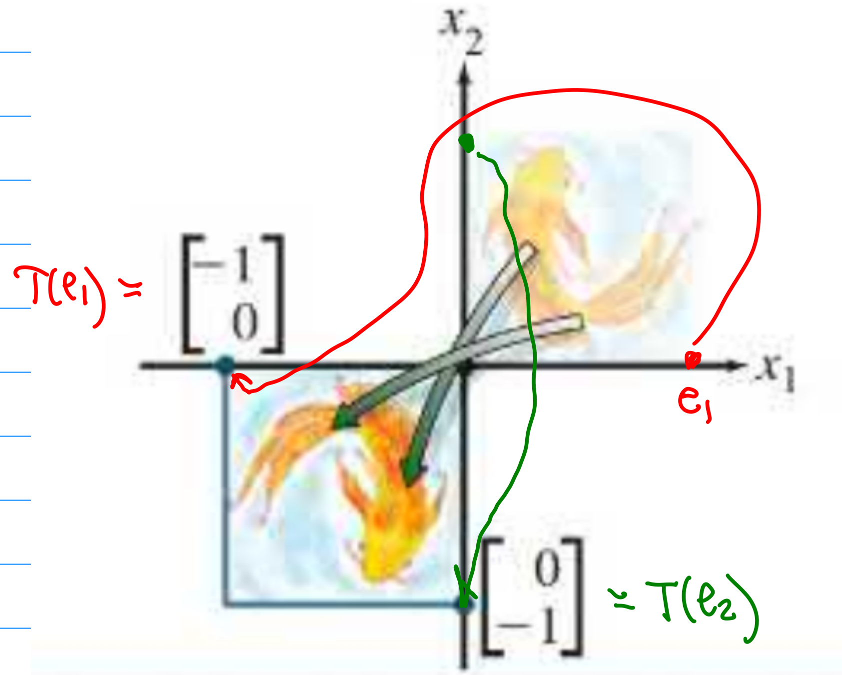
Reflection through the line $x_2 = x_1$



IGURE 2

Therefore, $A = \left[T(e_1) \middle| T(e_2) \right] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

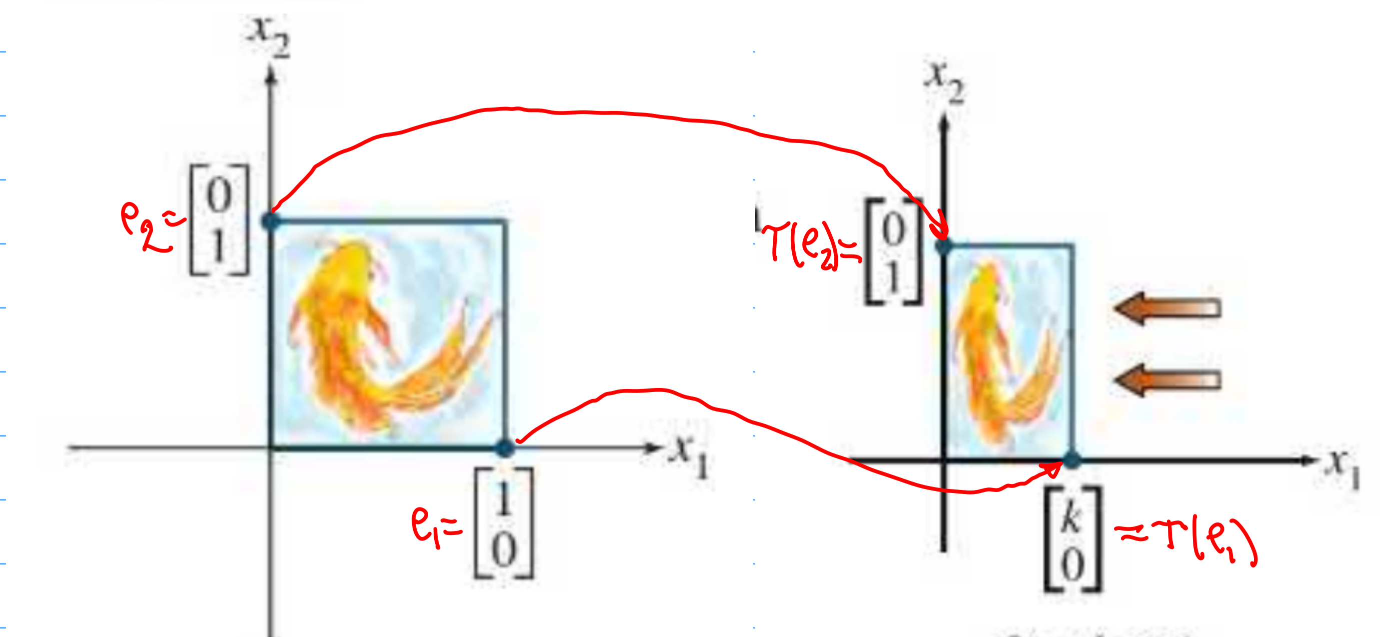
Reflection through the origin



Thus, the matrix is

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Horizontal contraction



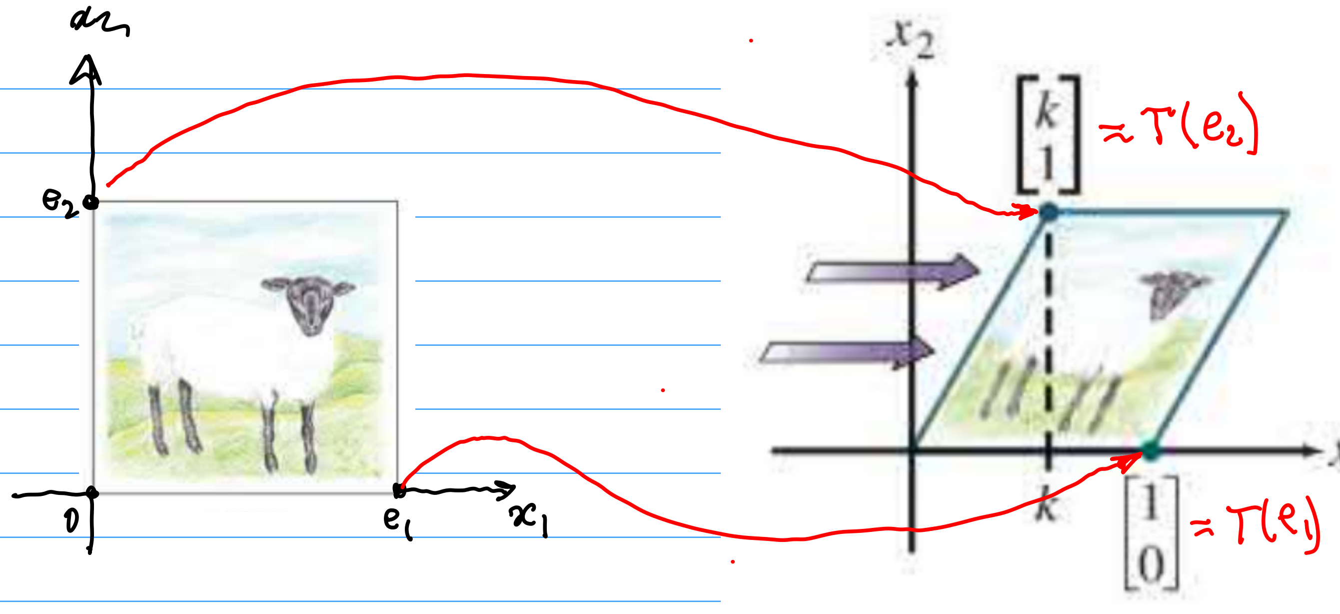
$0 < k < 1$

IGURE 2

$$\text{Matrix } A = \begin{bmatrix} T(e_1) \mid T(e_2) \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$$

Note this corresponds to a rescaling row operation.
In fact all the row operations are linear transformations

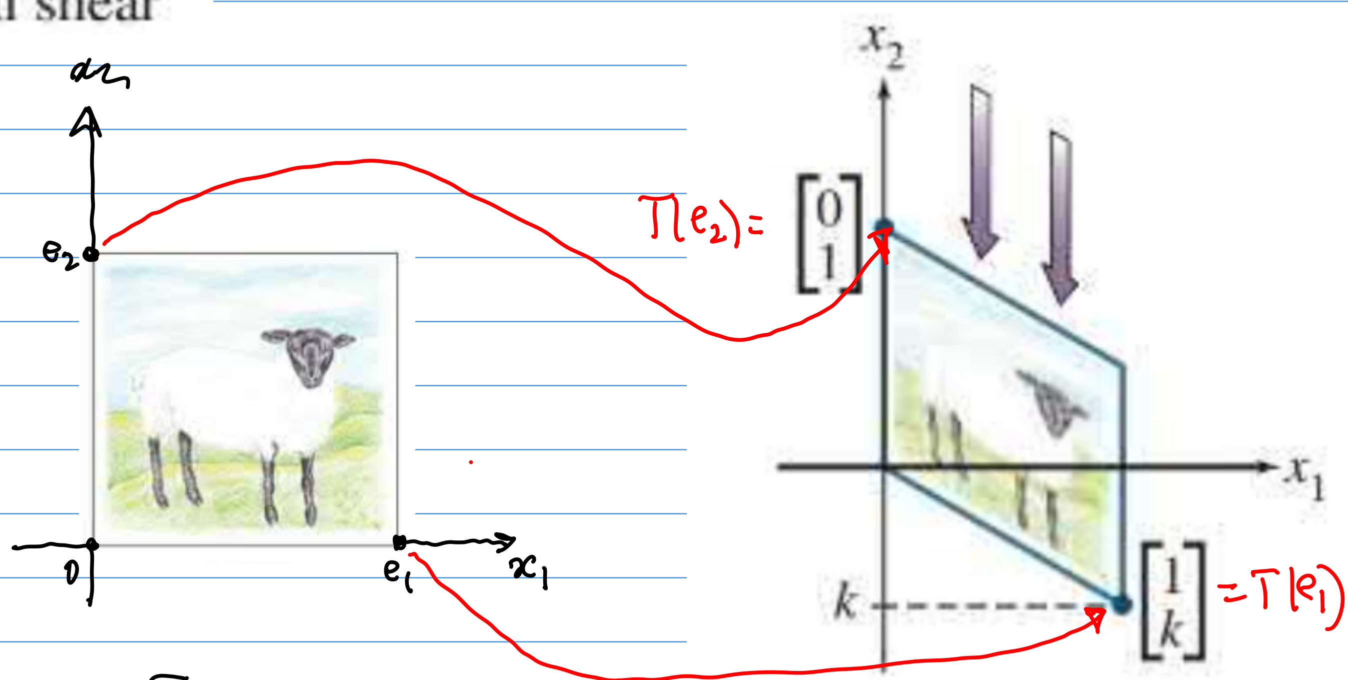
Horizontal shear



$k > 0$

Thus $A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

Vertical shear

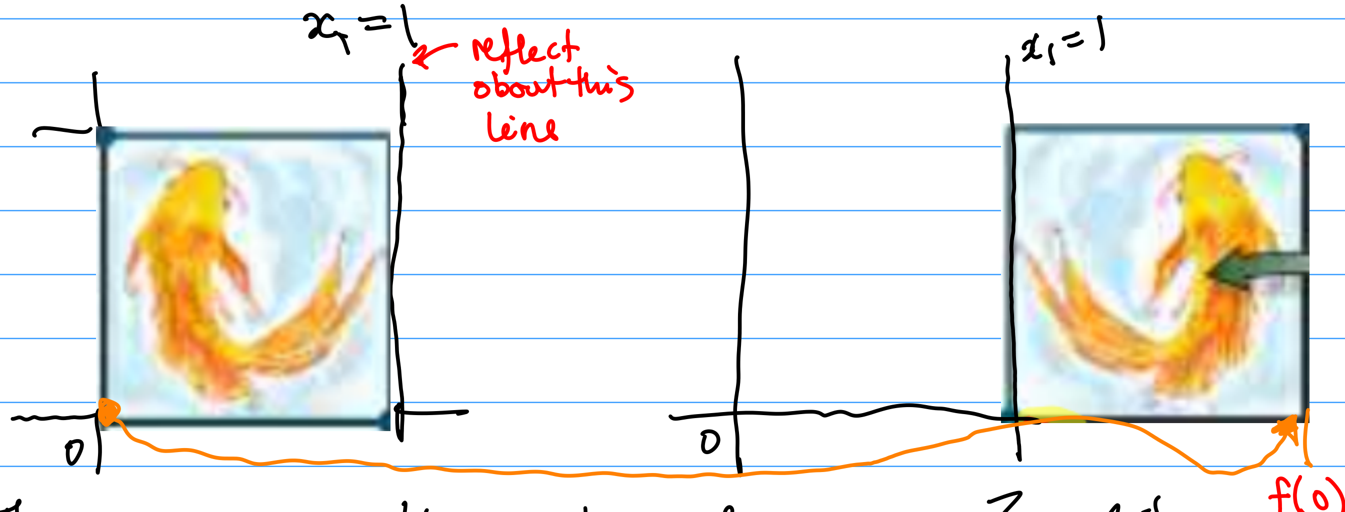


$k < 0$

$$A = \begin{bmatrix} T(e_1) \mid T(e_2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$$

Note the shearing operation correspond to elimination steps in the row operations

Note, the reflection about the vertical line at



This is not a linear transformation? Why

If $T(x) = Ax$ but $A0 = 0$

Note it is a reflection about x_2 axis plus a translation.

Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let A be the standard matrix for T. Then:

a. T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if the columns of A span $\mathbb{R}^m$;

b. T is one-to-one if and only if the columns of A are linearly independent.

b
- $Ax = 0$ has only the solution $x = 0$
- means A has No free variables

what about this, when do the columns span?

This means for every $b \in \mathbb{R}^m$ you can solve $Ax = b$, and find an x so that $T(x) = b$.

must be consistent

To solve write augmented matrix

$[A \mid b]$

Echelon form

$$\left[\begin{array}{ccccc|c} * & 0 & 0 & 0 & 0 & ? \\ 0 & 0 & * & 0 & 0 & ? \\ 0 & 0 & 0 & * & 0 & \vdots \\ 0 & 0 & 0 & 0 & 0 & ? \end{array}\right]$$

maybe not consistent, then A didn't span $\mathbb{R}^m$.

$$\left[\begin{array}{ccccc|c} * & 0 & 0 & 0 & 0 & ? \\ 0 & 0 & * & 0 & 0 & ? \\ 0 & 0 & 0 & * & 0 & \vdots \\ 0 & 0 & 0 & 0 & * & ? \end{array}\right]$$

There is a pivot in every row

That means can always solve $Ax = b$

So that means

$T : \mathbb{R}^n \to \mathbb{R}^m$

is onto.

Next time ...

2.1 Matrix Operations

Let A, B, and C be matrices of the same size, and let r and s be scalars.

a. $A + B = B + A$

b. $(A + B) + C = A + (B + C)$

c. $A + 0 = A$

d. $r(A + B) = rA + rB$

e. $(r + s)A = rA + sA$

f. $r(sA) = (rs)A$

ROW–COLUMN RULE FOR COMPUTING AB

If the product AB is defined, then the entry in row i and column j of AB is the sum of the products of corresponding entries from row i of A and column j of B. If $(AB)_{ij}$ denotes the (i, j)-entry in AB, and if A is an $m \times n$ matrix, then

$$(AB)_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

Why so complicated?

$f(x) = Ax$ $\qquad$ $g(x) = Bx$

composition of functions

Consider $f \circ g$ what's that? $\qquad$ AB

Note the composition of linear functions is again a linear function.. The Question is, what matrix corresponds to that composition?