

Let's suppose we've got a similarity $A = PDP^{-1}$

from last time

$$A = PDP^{-1}$$

$$[T]_{\beta} = \left[[T(b_1)]_{\beta} \mid [T(b_2)]_{\beta} \mid \dots \mid [T(b_n)]_{\beta} \right]$$

$$= \left[[Ab_1]_{\beta} \mid [Ab_2]_{\beta} \mid \dots \mid [Ab_n]_{\beta} \right]$$

$$[Ab_1]_{\beta} = r_1 \quad \text{means} \quad Ab_1 = Pr_1 \quad \text{so} \quad r_1 = P^{-1}Ab_1$$

$$[Ab_1]_{\beta} = P^{-1}Ab_1$$

recall

$$[x]_{\beta} = r \quad \text{means} \quad x = Pr$$

$$[T]_{\beta} = \left[P^{-1}Ab_1 \mid P^{-1}Ab_2 \mid \dots \mid P^{-1}Ab_n \right]$$

now factor things out

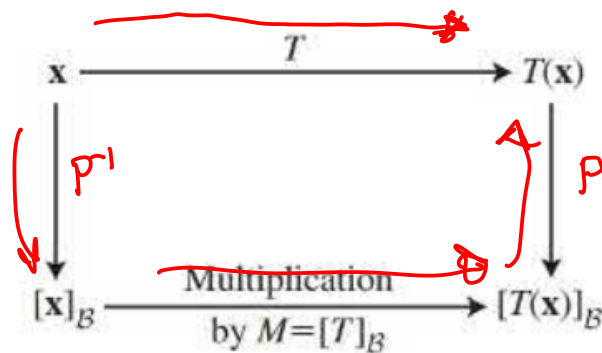
$$= P^{-1} \left[Ab_1 \mid Ab_2 \mid \dots \mid Ab_n \right]$$

recall

$$A = PDP^{-1}$$

$$= P^{-1}A[b_1 \mid b_2 \mid \dots \mid b_n] = P^{-1}AP = P^{-1}PDP^{-1}P = D$$

A diagram...



5.5 Complex Eigenvalues

Suppose $A \in \mathbb{R}^{n \times n}$. Then the eigenvalues of A are the roots of the characteristic polynomial $\chi_A(\lambda) = \det(A - \lambda I)$.

Sometimes those roots are complex... even though the coefficients of the polynomial are real...

In \mathbb{R}^2 we have $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$\chi_A(\lambda) = \det \begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix} = (a_{11} - \lambda)(a_{22} - \lambda) - a_{12}a_{21}$$

$$= \lambda^2 - (a_{11} + a_{22})\lambda + a_{11}a_{22} - a_{12}a_{21}$$

Eigenvalues ... use quadratic formula

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1, \quad b = -(a_{11} + a_{22})$$

$$c = a_{11}a_{22} - a_{12}a_{21}$$

when is $b^2 - 4ac < 0$?

$$(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})$$

$$\equiv (a_{11} + a_{22})^2 - 4a_{11}a_{22} + 4a_{12}a_{21} < 0$$

Claim: if $a_{12} = a_{21}$ then the above is never negative..

... next time ...