Math 330: Exam 2 Version A Sample Exam

This is a closed-book closed-notes no-calculator-allowed in-class exam. Efforts have been made to keep the arithmetic simple. If it turns out to be complicated, that's either because I made a mistake or you did. In either case, do the best you can and check your work where possible. While getting the right answer is nice, this is not an arithmetic test. It's more important to clearly explain what you did and what you know.

1. Indicate in writing that you have understood the requirement to work independently by writing "I have worked independently on this quiz" followed by your signature as the answer to this question.

2. Consider the matrix A with reduced row eschelon form R where

$$A = \begin{bmatrix} -2 & \frac{1}{3} & -2 & -\frac{1}{3} & -\frac{5}{2} \\ \frac{2}{3} & \frac{8}{9} & \frac{5}{3} & \frac{28}{9} & \frac{23}{6} \\ -4 & -\frac{1}{3} & -5 & -\frac{11}{3} & -\frac{19}{2} \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & 0 & \frac{7}{6} & \frac{2}{3} & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(i) Find a basis for $\mathrm{Col}(A)$.

(ii) Find a basis for $\mathrm{Nul}(A)$.

3. Answer the following true false questions:

(i) $\det(A+B) = \det(A) + \det(B)$.

(A) True

(B) False

(ii) The determinant of A is the product of the diagonal entries in A.

(A) True

(B) False

(iii) If the real matrix A has two complex eigenvalues, then it also has two linearly independent real eigenvectors.

(A) True

(B) False

(iv) If $A \in \mathbf{R}^{n\times n}$ satisfies $A^T = A$ then all the eigenvalues of A must be real.

(A) True

(B) False

4. Suppose that all the entries in A are integers and $\det(A) = 1$. Explain why all the entires in A^{-1} are integers.

5. What is the rank of a 4×5 matrix whose null space is three dimensional?

6. Find $\det(A)$, $\det(B)$ and $\det(AB)$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

7. Find the eigenvalues and eigenvectors of the matrix A where

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}.$$

8. The matrix A given by

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

has eigenvalues λ_i and eigenvectors x_i given by

$$\lambda_1 = 2, \quad x_1 = \begin{bmatrix} 2/3 \\ 1 \\ 1 \end{bmatrix}, \qquad \lambda_2 = 3, \quad x_2 = \begin{bmatrix} 1/4 \\ 3/4 \\ 1 \end{bmatrix}, \qquad \lambda_3 = 1, \quad x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Find an invertible matrix S and a diagonal matrix D such that $A = SDS^{-1}$.

(i) What is D?

(ii) What is S?